JEPPIAAR INSTITUTE OF TECHNOLOGY

QUESTION BANK

FIRST YEAR – 2nd SEMESTER

DEPARTMENT OF SCIENCE AND HUMANITIES

Syllabus

MA8251 ` MATHEMATICS – II L T P C

3 1 0 4

OBJECTIVES:

 To make the student acquire sound knowledge of techniques in solving ordinary differential equations that model engineering problems.

 To acquaint the student with the concepts of vector calculus, needed for problems in all engineering disciplines.

 To develop an understanding of the standard techniques of complex variable theory so as to enable the student to apply them with confidence, in application areas such as heat conduction, elasticity, fluid dynamics and flow the of electric current.

 To make the student appreciate the purpose of using transforms to create a new domain in which it is easier to handle the problem that is being investigated.

UNIT I MATRICES 9+3

Eigenvalues and Eigenvectors of a real matrix - Characteristic equation - Properties of eigenvalues and eigenvectors - Statement and applications of Cayley-Hamilton Theorem - Diagonalization of matrices - Reduction of a quadratic form to canonical form by orthogonal transformation - Nature of quadraticforms.

UNIT II VECTOR CALCULUS 9+3

Gradient, divergence and curl – Directional derivative – Irrotational and solenoidal vector fields – Vector integration – Green's theorem in a plane, Gauss divergence theorem and Stokes' theorem(excluding proofs) – Simple applications involving cubes and rectangular parallelopipeds.

UNIT III ANALYTIC FUNCTIONS 9+3

Functions of a complex variable – Analytic functions: Necessary conditions – Cauchy-Riemann equations and sufficient conditions (excluding proofs) – Harmonic and orthogonal properties of analytic function – Harmonic conjugate – Construction of analytic functions – Conformal mapping: $w = z+k$, kz, $1/z$, z^2, e^z and bilinear transformation.

UNIT IV COMPLEX INTEGRATION 9+3

Complex integration – Statement and applications of Cauchy's integral theorem and Cauchy's integral formula – Taylor's and Laurent's series expansions – Singular points – Residues – Cauchy's residue theorem – Evaluation of real definite integrals as contour integrals around unit circle and semi-circle (excluding poles on the real axis).

UNIT V LAPLACE TRANSFORM 9+3

Laplace transform – Sufficient condition for existence – Transform of elementary functions – Basic properties – Transforms of derivatives and integrals of functions - Derivatives and integrals of transforms - Transforms of unit step function and impulse functions – Transform of periodic functions. Inverse Laplace transform -Statement of Convolution theorem – Initial and final value theorems – Solution of linear ODE of second order with constant coefficients using Laplace transformation techniques.

TOTAL: 60 PERIODS

TEXT BOOKS:

1. Bali N. P and Manish Goyal, "A Text book of Engineering Mathematics", Eighth Edition, Laxmi Publications Pvt Ltd.,(2011).
2. Grewal. B.S, "Higher Engineering Mathematics", 41 st Edition, Khanna Publications, Delhi, (2011).

REFERENCES:

1. Dass, H.K., and Er. Rajnish Verma," Higher Engineering Mathematics", S. Chand Private Ltd., (2011)
2. Glyn James, "Advanced Modern Engineering Mathematics", 3rd Edition, Pearson Education, (2012).
3. Peter V. O'Neil," Advanced Engineering Mathematics", 7th Edition, Cengage learning, (2012).
4. Ramana B.V, "Higher Engineering Mathematics", Tata McGraw Hill Publishing Company, New Delhi, (2008).

TABLE OF CONTENT

Subject Code / Subject Name: MA8251/ ENGINEERING MATHEMATICS-II
Year/Semester: I /II

	UNIT-I MATRICES
	Eigen values and Eigenvectors of a real matrix – Characteristic equation – Properties of Eigen values and Eigenvectors – Cayley-Hamilton theorem – Diagonalization of matrices – Reduction of a quadratic form to canonical form by orthogonal transformation – Nature of quadratic forms.
Q.No.	**PART-A**
1	**State Cayley Hamilton theorem and give its two uses.** **(NOV/DEC 2015)(MAY/JUNE 2012)** BTL1 Every square matrix satisfies its own characteristic equation. It is used to calculate i. The positive integral powers ii. The inverse of a square matrix.
2	**If $\lambda_1, \lambda_2, \ldots\lambda_n$ are Eigen values of a matrix A then show that $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \ldots \frac{1}{\lambda_n}$ are Eigen values of A^{-1}.** BTL2 If λ_i and X_i are corresponding Eigen value and Eigen vector of A where i=1,2...,n. $AX_i =_i X_i A^{-1}(AX_i) = A^{-1}(\lambda_i X_i)$ $\Rightarrow IX_i = \lambda_i A^{-1} X_i$ $\Rightarrow X_i = \lambda_i A^{-1} X_i$ $\Rightarrow A^{-1} Xi = 1/\lambda_i X_i$ $\Rightarrow A^{-1} = 1/\lambda_i$ $\therefore 1/\lambda_i$ is an Eigen values of A^{-1}
3	If $\lambda_1,\lambda_2,\ldots \lambda n$ are Eigen values of an n x n matrix A then show that $\lambda_1^3, \lambda_2^3 \ldots \lambda_n^3$ are Eigen values of A^3. BTL2 Let λ be Eigen value of A and let X be Eigen vector of A. $\therefore \quad AX = \lambda X$

	$A^2X = A\lambda X$ $= \lambda\,(AX)$ $= \lambda\,(\lambda X)$ $= \lambda^2 X$ $\therefore A^2 = \lambda$ Similarly, $A^3X = \lambda^3 X \Rightarrow A^3 = \lambda^3$ $\therefore \lambda^3$ is an Eigen value of A^3.
4	**Two Eigen values of A=$\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ are equal and are $\frac{1}{5}$ times to the third. Find them.** (NOV/DEC 2014) BTL1 Let $\lambda_1, \lambda_2, \lambda_3$ be Eigen values of A. Given $\lambda_1 = \lambda_2 = \frac{1}{5}\lambda_3$ We know sum of Eigen values = sum of diagonal elements $\lambda_1 + \lambda_2 + \lambda_3 = 7$ $\frac{1}{5}\lambda_3 + \frac{1}{5}\lambda_3 + \lambda_3 = 7$ $\frac{7}{5}\lambda_3 = 7$ $\therefore \lambda_3 = 5$ $\therefore \lambda_1 = \lambda_2 = 1.$
5	**Find the Eigen values of A^2 given A=$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & -7 \\ 0 & 0 & 3 \end{pmatrix}$.Also find A^3, A^{-1}, $2A^2$.** BTL1 We know the Eigen values of a triangular matrix are just the diagonal elements. Here given matrix is a upper triangular matrix $\therefore$ Eigen values of A are 1,2,3. We know that "if $\lambda_1, \lambda_2, \ldots\lambda_n$ are Eigen values of a matrix A, then $\lambda_1^m, \lambda_2^m, \ldots \lambda_n^m$ are Eigen values of A^m." $\therefore$ Eigen values of A^2 are 1,4,9. $\therefore$ Eigen values of A^3 are 1,8,27. We know that if $\lambda_1, \lambda_2, \ldots\lambda_n$ are Eigen

	values of A then k λ_1, kλ_2, …kλ_n are Eigen values of KA $\therefore$ Eigen values of $2A^2$ are 2,8,18
6	**If A is an orthogonal matrix Show that A^{-1} is also orthogonal** BTL2 Let A be orthogonal matrix i.e. $A^T = A^{-1}$ Let $A^T = A^{-1} = B$ $B^T = (A^{-1})^T = (A^T)^{-1} = B^{-1}$ Therefore B is orthogonal. i.e. A^{-1} is an orthogonal matrix.
7	**Prove that the product of 2 orthogonal matrices is an orthogonal matrix.** BTL5 Let A be an n^{th} order orthogonal matrix. $\therefore \quad AA' = A'A = I$ Let B be an n^{th} order orthogonal matrix. $BB' = B'B = I$ Now $(AB)(AB)' = AB\,B'A'$ $= AIA'$ $= AA'$ $= I$ Now $(AB)'(AB) = B'A'AB$ $= B'IB$ $= B'B$ $= I$ Since $(AB)(AB)' = (AB)'(AB) = I$. AB is orthogonal matrix.
8	**If 1 and 2 are Eigen values of a 2 x2 matrix A, what are the Eigen values of A^2 and A^{-1}.** BTL1 Eigen values of A^2 are 1 and 4 Eigen values of A^{-1} are 1 and $\frac{1}{2}$.
9	**If 2, 3 are the Eigen value of** $A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ b & 0 & 2 \end{pmatrix}$ **then find the value of b?** **(NOV/DEC 2013)** BTL1

	Given Eigen values are $\lambda_1 = 2, \lambda_3 = 3$ Sum of the Eigen values = Sum of the main diagonal elements $\lambda_1 + \lambda_2 + \lambda_3 = 6$ $2 + 3 + \lambda_3 = 6$ $5 + \lambda_3 = 6$ $\lambda_3 = 1$ Product of the Eigen value $= \lvert A \rvert$ $(2)(3)(1) = 8 - 2b$ $6 = 8 - 2b$ $b = 1$
10	**If the sum of two Eigen values and trace of a 3 x 3 matrix A are equal, find the value of $\lvert A \rvert$.** BTL1 Let $\lambda_1, \lambda_2, \lambda_3$ be the Eigen values of A. Then we have $\lambda_1 + \lambda_2$ = trace of A $\Rightarrow \lambda_1 + \lambda_2 = \lambda_1 + \lambda_2 + \lambda_3 \Rightarrow \lambda_3 = 0$. Hence $\lvert A \rvert$ = product of Eigen values = $\lambda_1 \lambda_2 \lambda_3 = 0$
11	**For a given matrix A of order 3, $\lvert A \rvert$ = 32 and two of its Eigen values are 8 and 2. Find the sum of the Eigen values** BTL1 Given Eigen value be $\lambda_1 = 8, \lambda_2 = 2$. Then (8)(2)($\lambda_3$) = $\lvert A \rvert$ = 32 $\Rightarrow \lambda_3 = 2$ Let the third Eigen value be $\lambda_3 = 2$ Hence the sum of the Eigen values = $\lambda_1 + \lambda_2 + \lambda_3 = 8 + 2 + 2 = 12$
12	**Find the sum and product of the Eigen values of the square matrix** $A = \begin{pmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{pmatrix}$ **(NOV/DEC 2010)** BTL1 Sum of the Eigen values = sum of the main diagonal elements = 8+5+2=15 Product of the Eigen values = $\lvert A \rvert = 8(10-63) - 1(6-28) + 6(27-20) = -360$
13	**Find the sum of the Eigen values of 2A if** $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ BTL1

	If λ_1, λ_2, λ_3 are the Eigen values of A, then $\lambda_1 + \lambda_2 + \lambda_3 = 18$. We know that $2\lambda_1$, $2\lambda_2$, $2\lambda_3$ are the Eigen values of 2A. Therefore the sum of Eigen values of 2A = $2(\lambda_1 + \lambda_2 + \lambda_3) = 2(18) = 36$
14	**If the Eigen value of A are 3x3 are 2,3 and 1, then find the Eigen values of adjA. (NOV/DEC 2003)** BTL1 The Eigen values of are 2,3,1 The Eigen value of A^{-1} are $\frac{1}{2}, \frac{1}{3}, 1$ The product of Eigen values are $(2)(3)(1) = \|A\|$ $\therefore \|A\| = 6$ We know that $A^{-1} = \frac{1}{\|A\|} adjA$ $adjA = \|A\| A^{-1}$ The Eigen value of adjA are $(6)\left(\frac{1}{2}\right), (6)\left(\frac{1}{3}\right), (6)1$ $\Rightarrow 3, 2, 6$
15	**Find the sum of the squares of the Eigen values of** $A = \begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$ **(NOV/DEC 2016)** BTL1 A is a triangular matrix. Therefore the Eigen values of A are 3, 2 and 5. The sum of squares of the Eigen values of $A^2 = 3^2 + 2^2 + 5^2 = 9 + 4 + 25 = 3$
16	**Find the Eigen values of 2A – I, given** $A = \begin{pmatrix} -4 & 1 \\ 3 & -2 \end{pmatrix}$ BTL1 $2\boldsymbol{A} - \boldsymbol{I} = \begin{pmatrix} -8 & 2 \\ 6 & -4 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -9 & 2 \\ 6 & -5 \end{pmatrix}$ The Characteristic equation of 2A - I is given by

	$\lvert 2\boldsymbol{A}-\boldsymbol{I}-\lambda\boldsymbol{I}\rvert = 0 \Rightarrow \begin{vmatrix} -9-\lambda & 2 \\ 6 & -5-\lambda \end{vmatrix} = 0$ $\Rightarrow \lambda^2+14\lambda+33=(\lambda+11)(\lambda+3)=0$ $\Rightarrow \lambda=-3,-11$
17	**Prove that A and A^T have the same Eigen values** BTL5 $\lvert \boldsymbol{A}^T-\lambda\boldsymbol{I}\rvert = \lvert \boldsymbol{A}^T-(\lambda\boldsymbol{I})^T\rvert = \lvert(\boldsymbol{A}-\lambda\boldsymbol{I})^T\rvert = \lvert \boldsymbol{A}-\lambda\boldsymbol{I}\rvert$ $\Rightarrow$ A and A^T have the same characteristic equation and hence they have the same Eigen values.
18	**Prove that Similar matrices have the same characteristic roots** BTL5 Let A and B be two similar matrices, then there exists a matrix P such that B = P^{-1}AP. Hence $\lvert \boldsymbol{B}-\lambda\boldsymbol{I}\rvert = \lvert \boldsymbol{P}^{-1}\boldsymbol{AP}-\boldsymbol{P}^{-1}\lambda\boldsymbol{IP}\rvert = \lvert \boldsymbol{P}^{-1}\rvert\lvert \boldsymbol{A}-\lambda\boldsymbol{I}\rvert\lvert \boldsymbol{P}\rvert = \lvert \boldsymbol{A}-\lambda\boldsymbol{I}\rvert\lvert \boldsymbol{PP}^{-1}\rvert$ $= \lvert \boldsymbol{A}-\lambda\boldsymbol{I}\rvert$ i.e., A and B have the same characteristic equation. Therefore, they have the same Characteristic roots.
19	**Is the matrix** $B=\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta` & 0 \\ 0 & 0 & 1 \end{pmatrix}$ **orthogonal? Justify.** BTL5 $\mathbf{BB}^T = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \mathbf{I}$ Similarly, B^{T}B = I. Hence B is orthogonal.
20	**Use Cayley-Hamilton theorem to find** $A^4-4A^3-5A^2+A+2I$ **where** $A=\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$**.** BTL3 $\lvert \mathbf{A}-\lambda\boldsymbol{I}\rvert = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2-4\lambda-5=0 \Rightarrow \boldsymbol{A}^2-4\boldsymbol{A}-5\boldsymbol{I}=0$ **(**By Cayley-Hamilton Theorem) $\Rightarrow \boldsymbol{A}^2(\boldsymbol{A}^2-4\boldsymbol{A}-5\boldsymbol{I})=0 \Rightarrow \boldsymbol{A}^4-4\boldsymbol{A}^3-5\boldsymbol{A}^2=0$ $\Rightarrow A^4-4A^3-5A^2+A+2I = 0+A+2I = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}.$

21	**Can** $A=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ **be diagonalised? Why? (MAY/JUNE 2016)** BTL1 Yes. Even if the Eigen values of A are equal, namely 1, 1, it is possible to find two linearly independent Eigen vectors corresponding to the Eigen value 1.
22	**Find the matrix of the quadratic from** $2x^2+2y^2+3z^2+2xy-4xz-4yz$ BTL1 The required matrix $A=\begin{bmatrix} coeff\ x^2 & \frac{1}{2}coeff\ \ xy & \frac{1}{2}coeff\ xz \\ \frac{1}{2}coeff\ \ yx & coeff\ \ y^2 & \frac{1}{2}coeff\ yz \\ \frac{1}{2}coeff\ zx & \frac{1}{2}coeff\ \ zy & coeff\ z^2 \end{bmatrix}$ $A=\begin{pmatrix} 2 & 1 & -2 \\ 1 & 2 & -2 \\ -2 & -2 & 3 \end{pmatrix}$
23	**Find the nature of the quadratic form** $x_1^2+2x_2^2+x_3^2-2x_1x_2+2x_2x_3$ BTL1 **(MAY/JUNE 2010)** $A=\begin{bmatrix} coeffx_1^2 & \frac{1}{2}coeffx_1x_2 & \frac{1}{2}coeffx_1x_3 \\ \frac{1}{2}coeffx_2x_1 & coeffx_2^2 & \frac{1}{2}coeffx_2x_3 \\ \frac{1}{2}coeffx_3x_1 & \frac{1}{2}coeffx_3x_2 & coeffx_3^2 \end{bmatrix}$ $D_1=\begin{vmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{vmatrix}=\|a_{11}\|=1$ $D_2=\begin{vmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{vmatrix}=\begin{vmatrix} 1 & -1 \\ -1 & 2 \end{vmatrix}=2-1=1$ $D_3=\|A\|=1$ The nature positive definite since all are positive values.

24	**Write down the matrix corresponding to the quadratic form** $x^2+y^2+z^2+2zx+4\sqrt{2}yz$ BTL1 The required matrix $A=\begin{bmatrix} coeff\ x^2 & \frac{1}{2}coeff\ xy & \frac{1}{2}coeff\ xz \\ \frac{1}{2}coeff\ yx & coeff\ y^2 & \frac{1}{2}coeff\ yz \\ \frac{1}{2}coeff\ zx & \frac{1}{2}coeff\ zy & coeff\ z^2 \end{bmatrix}$ $A=\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2\sqrt{2} \\ 1 & 2\sqrt{2} & 1 \end{pmatrix}$
25	**Write down the Quadratic Form corresponding to the matrix** $A=\begin{pmatrix} 2 & 1 & -2 \\ 1 & 2 & -2 \\ -2 & -2 & 3 \end{pmatrix}$ BTL1 The Quadratic Form of the matrix is $2x^2+2y^2+3z^2+2xy-4yz-4zx$
26	**Define index and signature of a quadratic form. Find the index and signature of the quadratic form** $x_1^2+2x_2^2-3x_3^2$. BTL1 The number (p) of positive terms in the canonical form of a QF is called the index of the QF. The number of positive terms minus the number of negative terms is called the signature of the QF Index = 2 , Signature = 1
	Find the constant 'a' and 'b' such that the matrix $A=\begin{pmatrix} a & 4 \\ 1 & b \end{pmatrix}$ has 3 and -2 as eigen values. BTL1 Give the Eigen values are 3 and -2 Sum of the Eigen value of A are 'a' and 'b' Sum of the Eigen value $a+b=3-2=1$ $\therefore a+b=1$……………………….(1)

27	Product of the Eigen value $3(-2)=-6$ Product of the Eigen value of A are $\|A\|=ab-4$ $\therefore ab-4=-6$ $ab=-2$.....................(2) $(1)\Rightarrow\ b=1-a$ $(2)\Rightarrow\ ab=-2$ $a(1-a)=-2$ $a^2-a-2=0$ $(a-2)(a+1)=0 \quad \therefore a=2 \,\&\, a=-1$ $when\ \ a=2\ then\ b=-1$ $when\ \ a=-1\ then\ b=2$ $\therefore a=2, b=-1\ or\ a=-1, b=2$
28	**Find the Eigen values of 3A+2I , where** $A=\begin{pmatrix}5 & 4\\0 & 3\end{pmatrix}$ **(MAY/JUNE 2007)** BTL1 The Eigen values of A are 5 and 2 The Eigen values of 3A+2I are 3(5)+2 and 3(2)+2 The Eigen values of 3A+2I are 17 and 8
	PART-B
1.	**Find the Eigen values and Eigen vectors of** $\begin{pmatrix}2 & 2 & 0\\2 & 1 & 1\\-7 & 2 & -3\end{pmatrix}$ **(8 M)** BTL1 **Answer : Refer Page No.1.8- Dr.M.CHANDRASEKAR** • The Eigen values are $\lambda=-4,1,3$ **(2 M)** • Eigen vectors $X_1=\begin{bmatrix}1\\-3\\13\end{bmatrix}; X_2=\begin{bmatrix}2\\-1\\-4\end{bmatrix}; X_3=\begin{bmatrix}2\\1\\4\end{bmatrix}$ **(6 M)**
2.	**Find the Eigen values and Eigen vectors of** $\begin{pmatrix}1 & 0 & -1\\1 & 2 & 1\\2 & 2 & 3\end{pmatrix}$ **(DEC/JAN-2016 R-13) (8 M)** BTL1 **Answer : Refer Page No.1.10- Dr.M.CHANDRASEKAR**

	• The Eigen values are $\lambda = 1, 2, 3$ **(2 M)** • Eigen vectors $X_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}; X_2 = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}; X_3 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ **(6 M)**
3.	**Find the Eigen values and Eigen vectors of** $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ **(DEC/JAN-2014 R-13) (8 M)** BTL1 **Answer : Refer Page No.1.15- Dr.M.CHANDRASEKAR** • The Eigen values are $\lambda = 1, 1, 5$ **(2 M)** • Eigen vectors $X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}; X_2 = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}; X_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ **(6 M)**
4.	**Find the Eigen values and Eigen vectors of** $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ **(APR/MAY-2015 R-13) (8 M)** BTL1 **Answer : Refer Page No.1.17- Dr.M.CHANDRASEKAR** • The Eigen values are $\lambda = 2, 2, 8$ **(2 M)** • Eigen vectors $X_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}; X_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}; X_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ **(6 M)**
5.	**Verify Cayley-Hamilton theorem and hence find the inverse of the matrix** $\begin{pmatrix} 1 & 2 & -1 \\ 3 & -3 & 1 \\ 2 & 1 & -2 \end{pmatrix}$ **(DEC/JAN-2014 R-13) (8 M)** BTL3 **Answer : Refer Page No.1.45- Dr.M.CHANDRASEKAR** • The Characteristic Equation is $\lambda^3 + 4\lambda^2 - 4\lambda - 12 = 0$ **(2 M)**

	• For Proving $A^3+4A^2-4A-12I=0$ **(3 M)** • $A^{-1}=\frac{1}{12}\begin{pmatrix}5&3&-1\\8&0&-4\\9&3&-9\end{pmatrix}$ **(3 M)**
6.	**Verify Cayley-Hamilton theorem and hence find the inverse of the matrix** $\begin{pmatrix}1&0&3\\2&1&-1\\1&-1&1\end{pmatrix}$ **(DEC/JAN-2015 R-13) (8 M)** BTL3 **Answer : Refer Page No.1.47- Dr.M.CHANDRASEKAR** • The Characteristic Equation is $\lambda^3-3\lambda^2-\lambda+9=0$ **(2 M)** • For Proving $A^3-3A^2-A+9I=0$ **(3 M)** • $A^{-1}=\frac{-1}{9}\begin{pmatrix}0&-3&-3\\-3&-2&7\\-3&1&1\end{pmatrix}$ **(3 M)**
7.	**Use Cayley-Hamilton theorem to find the** A^4 **of the matrix** $\begin{pmatrix}2&-1&1\\0&1&2\\1&0&1\end{pmatrix}$ **(DEC/JAN-2016 R-13) (8 M)** BTL3 **Answer : Refer Page No.1.48- Dr.M.CHANDRASEKAR** • The Characteristic Equation is $\lambda^3-4\lambda^2+4\lambda+1=0$ **(2 M)** • $A^4=\begin{pmatrix}22&-19&-5\\24&-9&14\\19&-12&3\end{pmatrix}$ **(6 M)**
8.	**Use Cayley-Hamilton theorem to find** $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$ **of** $A=\begin{pmatrix}2&1&1\\0&1&0\\1&1&2\end{pmatrix}$ **(DEC/JAN-2006,APR/MAY 2005) (8 M)** BTL3 **Answer : Refer Page No.1.51- Dr.M.CHANDRASEKAR** • The Characteristic Equation is $\lambda^3-5\lambda^2+7\lambda-3=0$ **(2 M)**

	• For Proving $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I=A^2+A+I$ **(3 M)** • $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I=\begin{pmatrix}8&5&5\\0&3&0\\5&5&8\end{pmatrix}$ **(3 M)**
9.	**Diagonalize** $A=\begin{pmatrix}8&-6&2\\-6&7&-4\\2&-4&3\end{pmatrix}$ **by means of orthogonal transformation (12 M)** BTL1 **Answer : Refer Page No.1.72- Dr.M.CHANDRASEKAR** • The Eigen values are $\lambda=0,3,15$ **(2 M)** • Eigen vectors $X_1=\begin{bmatrix}1\\2\\2\end{bmatrix};X_2=\begin{bmatrix}2\\1\\-2\end{bmatrix};X_3=\begin{bmatrix}2\\-2\\1\end{bmatrix}$ **(4 M)** • $D=N^TAN=\begin{pmatrix}0&0&0\\0&3&0\\0&0&15\end{pmatrix}$ **(6 M)**
10.	**Diagonalize** $A=\begin{pmatrix}3&1&1\\1&3&-1\\1&-1&3\end{pmatrix}$ **by means of orthogonal transformation. (12 M)** BTL1 **Answer : Refer Page No.1.77- Dr.M.CHANDRASEKAR** • The Eigen values are $\lambda=1,4,4$ **(2 M)** • Eigen vectors $X_1=\begin{bmatrix}-1\\1\\1\end{bmatrix};X_2=\begin{bmatrix}1\\1\\0\end{bmatrix};X_3=\begin{bmatrix}-1\\1\\-2\end{bmatrix}$ **(4 M)** • $D=N^TAN=\begin{pmatrix}1&0&0\\0&4&0\\0&0&4\end{pmatrix}$ **(6 M)**

11. **Diagonalize** $A=\begin{pmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{pmatrix}$ **by means of orthogonal transformation.** BTL1
(DEC/JAN-2015 R-13) (12 M)
Answer : Refer Page No.1.87- Dr.M.CHANDRASEKAR

- The Eigen values are $\lambda=2,2,8$ **(2 M)**
- Eigen vectors $X_1=\begin{bmatrix}2\\-1\\1\end{bmatrix};X_2=\begin{bmatrix}1\\1\\-1\end{bmatrix};X_3=\begin{bmatrix}0\\1\\1\end{bmatrix}$ **(4 M)**
- $D=N^TAN=\begin{pmatrix}8&0&0\\0&2&0\\0&0&2\end{pmatrix}$ **(6 M)**

12. **Reduce the quadratic form** $10x_1^2+2x_2^2+5x_3^2+6x_2x_3-10x_3x_1-4x_1x_2$ **to a canonical form. Discuss its nature. (16 M)** BTL1
Answer : Refer Page No.1.99- Dr.M.CHANDRASEKAR

- The Eigen values are $\lambda=0,3,14$ **(2 M)**
- Eigen vectors $X_1=\begin{bmatrix}1\\-5\\4\end{bmatrix};X_2=\begin{bmatrix}1\\1\\1\end{bmatrix};X_3=\begin{bmatrix}3\\-1\\-2\end{bmatrix}$ **(4 M)**
- $D=N^TAN=\begin{pmatrix}0&0&0\\0&3&0\\0&0&14\end{pmatrix}$ **(6 M)**
- Canonical form= $0y_1^2+3y_2^2+14y_3^2$ **(2 M)**
- Rank=2, Index=2, Signature=2; Nature = Positive Semi definite **(2 M)**

13. **Reduce the quadratic form** $6x_1^2+3x_2^2+3x_3^2-2x_2x_3+4x_3x_1-4x_1x_2$ **to a canonical form. Discuss its nature. (DEC/JAN-2016, JAN-2014 R-13) (16 M)** BTL1
Answer : Refer Page No.1.102- Dr.M.CHANDRASEKAR

- The Eigen values are $\lambda=2,2,8$ **(2 M)**

	• Eigen vectors $X_1=\begin{bmatrix}2\\-1\\1\end{bmatrix};X_2=\begin{bmatrix}1\\2\\0\end{bmatrix};X_3=\begin{bmatrix}2\\-1\\-5\end{bmatrix}$ **(4 M)** • $D=N^TAN=\begin{pmatrix}2&0&0\\0&2&0\\0&0&8\end{pmatrix}$ **(6 M)** • Canonical form= $2y_1^2+2y_2^2+8y_3^2$ **(2 M)** • Rank=3, Index=3, Signature=3; Nature = Positive definite **(2 M)**
14.	**Reduce the quadratic form** $6x_1^2+3x_2^2+3x_3^2-2x_2x_3+4x_3x_1-4x_1x_2$ **to a canonical form by orthogonal reduction. (16 M)** BTL1 **Answer : Refer Page No.1.104- Dr.M.CHANDRASEKAR** • The Eigen values are $\lambda=2,3,6$ **(2 M)** • Eigen vectors $X_1=\begin{bmatrix}1\\0\\-1\end{bmatrix};X_2=\begin{bmatrix}1\\1\\1\end{bmatrix};X_3=\begin{bmatrix}1\\-2\\1\end{bmatrix}$ **(4 M)** • $D=N^TAN=\begin{pmatrix}2&0&0\\0&3&0\\0&0&6\end{pmatrix}$ **(8 M)** • Canonical form= $2y_1^2+3y_2^2+6y_3^2$ **(2 M)**
15.	**Reduce the quadratic form** $x^2+5y^2+z^2+2xy+2yz+6zx$ **to a canonical form through an orthogonal transformation. (DEC/JAN-2015 R-13) (16 M)** BTL1 **Answer : Refer Page No.1.109- Dr.M.CHANDRASEKAR** • The Eigen values are $\lambda=-2,3,6$ **(2 M)** • Eigen vectors $X_1=\begin{bmatrix}-1\\0\\1\end{bmatrix};X_2=\begin{bmatrix}-1\\1\\-1\end{bmatrix};X_3=\begin{bmatrix}1\\2\\1\end{bmatrix}$ **(4 M)**

	• $D=N^TAN=\begin{pmatrix}-2&0&0\\0&3&0\\0&0&6\end{pmatrix}$ **(8 M)** • Canonical form= $-2y_1^2+3y_2^2+6y_3^2$ **(2 M)**
16.	**Reduce the quadratic form** $8x_1^2+7x_2^2+3x_3^2-8x_2x_3+4x_3x_1-12x_1x_2$ **to a canonical form by orthogonal reduction. (16 M)** BTL1 **Answer : Refer Page No.1.111- Dr.M.CHANDRASEKAR** • The Eigen values are $\lambda=0,3,15$ **(2 M)** • Eigen vectors $X_1=\begin{bmatrix}1\\2\\2\end{bmatrix};X_2=\begin{bmatrix}2\\1\\-2\end{bmatrix};X_3=\begin{bmatrix}2\\-2\\1\end{bmatrix}$ **(4 M)** • $D=N^TAN=\begin{pmatrix}0&0&0\\0&3&0\\0&0&15\end{pmatrix}$ **(8 M)** • Canonical form= $0y_1^2+3y_2^2+15y_3^2$ **(2 M)**
17.	**Reduce the quadratic form** $2x_1^2+5x_2^2+3x_3^2+4x_1x_2$ **to a canonical form by orthogonal reduction. (16 M)** BTL1 **Answer : Refer Page No.1.113- Dr.M.CHANDRASEKAR** • The Eigen values are $\lambda=1,3,6$ **(2 M)** • Eigen vectors $X_1=\begin{bmatrix}2\\-1\\0\end{bmatrix};X_2=\begin{bmatrix}0\\0\\1\end{bmatrix};X_3=\begin{bmatrix}1\\2\\0\end{bmatrix}$ **(4 M)** • $D=N^TAN=\begin{pmatrix}1&0&0\\0&3&0\\0&0&6\end{pmatrix}$ **(8 M)** • Canonical form= $1y_1^2+3y_2^2+6y_3^2$ **(2 M)**
18.	**Reduce the quadratic form** $x_1^2+2x_2^2+x_3^2+2x_2x_3-2x_1x_2$ **to a canonical form through orthogonal transformation and hence show that it is positive semi-definite. Also give a non-zero set of values** (x_1,x_2,x_3) **which makes this quadratic form zero (16 M)** BTL1

	Answer : Refer Page No.1.121- Dr.M.CHANDRASEKAR • The Eigen values are $\lambda = 0,1,3$ **(2 M)** • Eigen vectors $X_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}; X_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} X_3 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ **(4 M)** • $D = N^T AN = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ **(6 M)** • Canonical form= $0y_1^2 + 1y_2^2 + 3y_3^2$ **(2 M)** • $x_1 = 1, x_2 = 1, x_3 = -1$ which makes Q.F is zero **(1 M)** • For proving Positive Semi definite **(1 M)**
	UNIT-II VECTOR CALCULUS
	Gradient and directional derivative – Divergence and curl – Vector identities – Irrotational and Solenoidal vector fields – Line integral over a plane curve – Surface integral – Area of a curved surface – Volume integral – Green's, Gauss divergence and Stokes theorems – Verification and application in evaluating line, surface and volume integrals.
	PART-A
1	**State Stokes theorem. (DEC/JAN-2015)** BTL1 The surface integral of the normal component of the curl of a vector point function $\vec{F}$ over an open surface 'S' is equal to the line integral of the tangential component of $\vec{F}$ around the closed curve 'C' bounding 'S' $\int_C \vec{F}.\vec{dr} = \iint_S (\nabla \times \vec{F}).\ \hat{n}\, ds$
2	**State Gauss divergence theorem. (DEC/JAN-2013) (NOV/DEC-2015)** BTL1 The surface integral of the normal component of a vector function $\vec{F}$ over a closed surface S enclosing volume V is equal to the volume integral of the divergence of $\vec{F}$ taken throughout the volume V $\iint_S \vec{F}.\hat{n}\, ds = \iiint_V \nabla.\vec{F} dv$
3	**State Green's theorem. (DEC/JAN-2009) (NOV/DEC-2010)** BTL1

	If $u, v, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}$ are continuous and single valued functions in the region R enclosed by the curve C, then $\int_C udx + vdy = \iint_R \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dxdy$
4	**Find curl $\vec{F}$ if $\vec{F} = xy\vec{i} + yz\vec{j} + zx\vec{k}$.** BTL1 $curl\,\vec{F} = \nabla \times \vec{F}$ $= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xy & yz & zx \end{vmatrix} = \vec{i}(0-y) - \vec{j}(z-0) + \vec{k}(0-x)$ $= -y\vec{i} - z\vec{j} - x\vec{k} = -(y\vec{i} + z\vec{j} + x\vec{k})$
5	**Prove that $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ is irrotational.** BTL5 $\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & zx & xy \end{vmatrix} = \sum \vec{i}\left[\frac{\partial}{\partial y}(xy) - \frac{\partial}{\partial z}(zx)\right]$ $= \sum \vec{i}\,[x - x] = 0\vec{i} + 0\vec{j} + 0\vec{k} = \vec{0}$. Hence, $\vec{F}$ is irrotational.
6	**Is the position vector $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ irrotational? Justify. (DEC/JAN-2016)** BTL5 $\nabla \times \vec{r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & y & z \end{vmatrix}$ $= \vec{i}\left[\frac{\partial}{\partial y}(z) - \frac{\partial}{\partial z}(y)\right] - \vec{j}\left[\frac{\partial}{\partial x}(z) - \frac{\partial}{\partial z}(x)\right] + \vec{k}\left[\frac{\partial}{\partial x}(y) - \frac{\partial}{\partial y}(x)\right]$ $= 0\vec{i} + 0\vec{j} + 0\vec{k} = \vec{0}$. Hence, $\vec{r}$ is irrotational.
7	**Prove that $3x^2y\vec{i} + (yz - 3xy^2)\vec{j} - \frac{z^2}{2}\vec{k}$ is a solenoidal.** BTL5 $\nabla.\vec{F} = \frac{\partial}{\partial x}(3x^2y) + \frac{\partial}{\partial y}(yz - 3xy^2) + \frac{\partial}{\partial z}\left(-\frac{z^2}{2}\right)$ $= (6xy) + (z - 6xy) + (\frac{-2z}{2}) = 0$ $\therefore\ \vec{F}$ is Solenoidal.
8	**Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\vec{i} + (3xz + 2xy)\vec{j} + (3xy - 2xz + 2z)\vec{k}$ is both solenoidal and irrotational.** BTL2 $\nabla.\vec{F} = \frac{\partial}{\partial x}(y^2 - z^2 + 3yz - 2x) + \frac{\partial}{\partial y}(3xz + 2xy) + \frac{\partial}{\partial z}(3xy - 2xz + 2z)$

	$= (-2) + (2x) + (-2x + 2)$ $= 0$ $\therefore\ \vec{F}$ is Solenoidal. $\nabla \times \vec{F} =$ $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y^2 - z^2 + 3yz - 2x & 3xz + 2xy & 3xy - 2xz + 2z \end{vmatrix}$ $= \vec{i}\left[\frac{\partial}{\partial y}(3xy - 2xz + 2z) - \frac{\partial}{\partial z}(3xz + 2xy)\right]$ $-\vec{j}\left[\frac{\partial}{\partial x}(3xy - 2xz + 2z) - \frac{\partial}{\partial z}(y^2 - z^2 + 3yz - 2x)\right]$ $+\vec{k}\left[\frac{\partial}{\partial x}(3xz + 2xy) - \frac{\partial}{\partial y}(y^2 - z^2 + 3yz - 2x)\right]$ $= [3x - 3x]\vec{i} - [(3y - 2z) - (-2z + 3y)]\vec{j} + [(3z + 2y) - (2y + 3z)]\vec{k}$ $\nabla \times \vec{F} = 0\vec{i} + 0\vec{j} + 0\vec{k} = \vec{0}$ Hence, $\vec{F}$ is irrotational.
9	**Find α such that $\vec{F} = (3x - 2y + z)\vec{i} + (4x + \alpha y - z)\vec{j} + (x - y + 2z)\vec{k}$ is solenoidal.** BTL1 Given $\nabla.\vec{F} = 0$ $\frac{\partial}{\partial x}(3x - 2y + z) + \frac{\partial}{\partial y}(4x + \alpha y - z) + \frac{\partial}{\partial z}(x - y + 2z) = 0$ $3 + \alpha + 2 = 0$ $\alpha + 5 = 0 \quad \therefore \alpha = -5$
10	**Find the constants a, b, c so that $\vec{F} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational. (DEC/JAN-2012)** BTL1 $\nabla \times \vec{F} = \vec{0}$ $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x + 2y + az & bx - 3y - z & 4x + cy + 2z \end{vmatrix} = \vec{0}$ $\vec{i}[c + 1] - \vec{j}[4 - a] + \vec{k}[b - 2] = 0\vec{i} - 0\vec{j} + 0\vec{k}$ i.e., $c + 1 = 0, 4 - a = 0, b - 2 = 0$ $\therefore \boldsymbol{c = -1, a = 4, b = 2}$
11	**Prove that div $\vec{r} = 3$ and curl $\vec{r} = \vec{0}$. (DEC/JAN-2016) (NOV/DEC-2010)** BTL5 $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

	$\nabla.\vec{r} = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(z) = 1 + 1 + 1 = 3$ $\nabla \times \vec{r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & y & z \end{vmatrix}$ $= \vec{i}\left[\frac{\partial}{\partial y}(z) - \frac{\partial}{\partial z}(y)\right] - \vec{j}\left[\frac{\partial}{\partial x}(z) - \frac{\partial}{\partial z}(x)\right] + \vec{k}\left[\frac{\partial}{\partial x}(y) - \frac{\partial}{\partial y}(x)\right]$ $= \mathbf{0\vec{i} + 0\vec{j} + 0\vec{k} = \vec{0}}$
12	**Prove that curl $(\textbf{grad } \emptyset) = \vec{\mathbf{0}}$ (NOV/DEC-2008)** BTL5 $grad\ \emptyset = \nabla\emptyset$ $= \vec{i}\frac{\partial\emptyset}{\partial x} + \vec{j}\frac{\partial\emptyset}{\partial y} + \vec{k}\frac{\partial\emptyset}{\partial z}$ $curl\ (grad\ \emptyset) = \nabla \times (\nabla\emptyset)$ $= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \frac{\partial\emptyset}{\partial x} & \frac{\partial\emptyset}{\partial y} & \frac{\partial\emptyset}{\partial z} \end{vmatrix}$ $= \sum \vec{i}\left[\frac{\partial^2\emptyset}{\partial y\partial z} - \frac{\partial^2\emptyset}{\partial z\partial y}\right]$ $= \sum \overrightarrow{\vec{i}[0]}$ $(Since\ mixed\ partial\ derivatives\ are\ equal)$ $= 0\vec{i} + 0\vec{j} + 0\vec{k} = \vec{0}$
13	**In what direction from $(3, 1, -2)$ is the directional derivative of $\emptyset = x^2y^2z^4$ maximum? Find also the magnitude of this maximum.** BTL1 $\nabla\emptyset = \vec{i}\frac{\partial\emptyset}{\partial x} + \vec{j}\frac{\partial\emptyset}{\partial y} + \vec{k}\frac{\partial\emptyset}{\partial z}$ $= \vec{i}[2xy^2z^4] + \vec{j}[2x^2yz^4] + \vec{k}[4x^2y^2z^3]$ $\nabla\varphi_{(3,1,-2)} = \vec{i}\left[2(3)(1)(16)\right] + \vec{j}\left[2(9)(1)(16)\right] + \vec{k}\left[4(9)(1)(-8)\right]$ $= 96\vec{i} + 288\vec{j} - 288\vec{k}$ $= 96\left(\vec{i} + 3\vec{j} - 3\vec{k}\right)$ The directional derivative is maximum in the direction of $96\left(\vec{i} + 3\vec{j} - 3\vec{k}\right)$ Maximum value is $\lvert\nabla\varphi\rvert = \left\lvert 96\left(\vec{i} + 3\vec{j} - 3\vec{k}\right)\right\rvert$ $= \sqrt{92^2(1+9+9)}$ $= 96\sqrt{19}$
14	**Find the unit vector normal to the surface $x^2 + y^2 = z$ at $(1, -2, 5)$.** BTL1 Given $\phi = x^2 + y^2 - z$

	Unit normal vector $\hat{n} = \frac{\nabla\phi}{\lvert\nabla\phi\rvert}$ .... (1) $\nabla\phi = \vec{i}\frac{\partial\phi}{\partial x} + \vec{j}\frac{\partial\phi}{\partial y} + \vec{k}\frac{\partial\phi}{\partial z}$ $= \vec{i}[2x] + \vec{j}[2y] + \vec{k}[-1]$ $\nabla\phi_{(1,-2,5)} = \vec{i}[2] + \vec{j}[-4] + \vec{k}[-1]$ $= 2\vec{i} - 4\vec{j} - \vec{k}$ $\lvert\nabla\phi\rvert = \sqrt{2^2 + (-4)^2 + (-1)^2}$ $= \sqrt{4+16+1} \quad = \sqrt{21}$ $\therefore (1) \Rightarrow \hat{n} = \frac{2\vec{i} - 4\vec{j} - \vec{k}}{\sqrt{21}}$
15	**Find the greatest rate of increase of** $\boldsymbol{\emptyset = xyz^2}$ **at** $\boldsymbol{(1,0,3)}$ BTL1 $\nabla\emptyset = \vec{i}\frac{\partial\emptyset}{\partial x} + \vec{j}\frac{\partial\emptyset}{\partial y} + \vec{k}\frac{\partial\emptyset}{\partial z}$ $= \vec{i}[yz^2] + \vec{j}[xz^2] + \vec{k}[2xyz]$ $\nabla\emptyset_{\ (1,0,3)} = 0\vec{i} + 9\vec{j} + 0\vec{k}$ $\therefore$ Greatest rate of increase $= \lvert\boldsymbol{\nabla\emptyset}\rvert = \sqrt{\boldsymbol{9^2}} = \boldsymbol{9}$
16	**State the physical interpretation of the line integral.** $\int_A^B \vec{F}.\vec{dr}$ BTL1 Physically $\int_A^B \vec{F}.\overrightarrow{dr}$ denotes the total work done by the force $\vec{F}$, in displacing a particle from A to B along the curve C.
17	**Define Solenoidal vector function. If** $\vec{V} = (x+3y)\vec{i} + (y-2z)\vec{j} + (x+2\lambda z)\vec{k}$ **is Solenoidal, find the value of** λ**.** BTL1 If div $\vec{F}$= 0, then $\vec{F}$ is said to be Solenoidal vector. $\nabla.\vec{F} = 0$. $\nabla.\vec{V} = \frac{\partial}{\partial x}(x+3y) + \frac{\partial}{\partial y}(y-2z) + \frac{\partial}{\partial z}(x+2\lambda z)$ $= 1+1+2\lambda$ $= 2+2\lambda$ $\nabla.\vec{V} = 0$ $2+2\lambda = 0$ $\lambda = -1$
	Find grad $\boldsymbol{(r^n)}$ **where** $\boldsymbol{\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}}$ **and** $r = \lvert\vec{r}\rvert$**.** BTL1

18	We know that $\frac{\partial r}{\partial x}=\frac{x}{r}$, $\frac{\partial r}{\partial y}=\frac{y}{r}$, $\frac{\partial r}{\partial z}=\frac{z}{r}$ $grad\left(r^n\right)=\sum \vec{i}\frac{\partial r^n}{\partial x}$ $=\sum \vec{i}(nr^{n-1})\frac{\partial r}{\partial x}$ $=(nr^{n-2})\vec{r}$
19	**Find grad (r) and grad ($\frac{1}{r}$) where** $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$ **and** $r=\|\vec{r}\|$ BTL1 $\nabla\phi=\Sigma\vec{i}\frac{\partial\phi}{\partial x}=\frac{\Sigma x\vec{i}}{r}$ $=\frac{\vec{r}}{r}$ $grad(\frac{1}{r})=\Sigma\vec{i}\frac{\partial\left(\frac{1}{r}\right)}{\partial x}=\left(-\frac{1}{r^2}\right)\frac{\Sigma x\vec{i}}{r}$ $=\frac{-\vec{r}}{r^3}$
20	**Find the unit normal to the surface** $x^2+xy+z^2=4$ at $(1,-1,2)$. BTL1 $\hat{n}=\frac{\nabla\phi}{\|\nabla\phi\|}$ $\nabla\phi=\Sigma\vec{i}\frac{\partial\phi}{\partial x}$ *Given*: $x^2+xy+z^2=4$ $Point(1,-1,2)$ $\nabla\phi=\vec{i}+\vec{j}+4\vec{k}$ $\|\nabla\phi\|=\sqrt{1+1+16}=\sqrt{18}$ $\hat{n}=\frac{\vec{i}+\vec{j}+4\vec{k}}{3\sqrt{2}}$
21	**Prove by Green's theorem that the area bounded by a simple closed curve is** $\frac{1}{2}\int_c(xdy-ydx)$ BTL5

	By Green's theorem: $$\int_C u\,dx + v\,dy = \iint_R \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dxdy$$ $$u = \frac{-y}{2}, \mathrm{v} = \frac{v}{2} \Rightarrow \frac{\partial u}{\partial y} = \frac{-1}{2}, \frac{\partial v}{\partial x} = \frac{1}{2}$$ Given that $$\frac{1}{2}\int_C x\,dy - y\,dx = \iint_R \left(\frac{1}{2} + \frac{1}{2}\right) dxdy$$ $$= \iint_R dxdy.\ \ \text{which a area bounded by a simple closed curve}\ 'c'$$
22	**Find** $\nabla\left[\nabla.\left(\left(x^2 - yz\right)\vec{i} + \left(y^2 - xz\right)\vec{j} + \left(z^2 - xy\right)\vec{k}\right)\right]$ **at the point (1,-1,2).** BTL1 $$\nabla.\vec{F} = \frac{\partial}{\partial x}(x^2 - yz) + \frac{\partial}{\partial y}(\mathrm{y}^2 - xz) + \frac{\partial}{\partial z}(\mathrm{z}^2 - xy)$$ $$= 2x + 2y + 2z$$ $$\nabla.\vec{F}_{(1,-1,2)} = 2 - 2 + 4$$ $$= 4$$ $$Grad(\nabla.\vec{F}) = \nabla(\nabla.\vec{F})$$ $$= \vec{i}\frac{\partial}{\partial x}(2x) + \vec{j}\frac{\partial}{\partial y}(2y) + \vec{k}\frac{\partial}{\partial z}(2z)$$ $$= 2\vec{i} + 2\vec{j} + 2\vec{k}$$
23	**Find the directional directive of** $\phi(\mathrm{x,y,z}) = \mathrm{xy}^2 + yz^2$ **at the point (2,-1,1) in the direction of the vector** $\vec{i} + 2\vec{j} + 3\vec{k}$ **. (DEC/JAN-2014)** BTL1 Directional derivative(D.D)= $\nabla\phi.\frac{\vec{a}}{\lvert\vec{a}\rvert}$

	$Given:$ $\phi(x,y,z)=xy^2+z^2y,\quad \vec{a}=\vec{i}+2\vec{j}+3\vec{k}$ $\nabla\phi_{(1,-1,2)}=\vec{i}+2\vec{j}+4\vec{k},\quad |\vec{a}|=\sqrt{14}$ $D.D=(\vec{i}+2\vec{j}+4\vec{k}).\frac{(\vec{i}+2\vec{j}+3\vec{k})}{\sqrt{14}}$ $=\frac{17}{\sqrt{14}}.$
24	**If $\vec{F}$ is irrotational and C is closed curve then find the value of** $\int_c \vec{F}.d\vec{r}$ BTL1 By Stokes theorem $\int_c \vec{F}.d\vec{r}=\iint_s(\nabla \times \vec{F}).\hat{n}ds$ Since $\vec{F}$ is irrotational $\therefore \nabla \times \vec{F}=0$ $\int_c \vec{F}.d\vec{r}=\iint_s(\nabla \times \vec{F}).\hat{n}ds$ $=\iint_s 0.\hat{n}ds$ $=0$
25	**Prove that** $\nabla(\log r)=\frac{\vec{r}}{r^2}$. **(NOV/DEC-2014)** BTL5 we have $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$ and $r=|\vec{r}|=\sqrt{x^2+y^2+z^2}$ $r^2=x^2+y^2+z^2,\ \frac{\partial r}{\partial x}=\frac{x}{r},\ \frac{\partial r}{\partial y}=\frac{y}{r},\ \frac{\partial r}{\partial z}=\frac{z}{r}$ $\nabla(\log r)=\vec{i}\,\frac{\partial(\log r)}{\partial x}+\vec{j}\,\frac{\partial(\log r)}{\partial y}+\vec{k}\,\frac{\partial(\log r)}{\partial z}$ $=\vec{i}\left(\frac{1}{r}\frac{\partial r}{\partial x}\right)+\vec{j}\left(\frac{1}{r}\frac{\partial r}{\partial y}\right)+\vec{k}\left(\frac{1}{r}\frac{\partial r}{\partial z}\right)$ $=\frac{1}{r}\left[\frac{x}{r}\vec{i}+\frac{y}{r}\vec{j}+\frac{z}{r}\vec{k}\right]$ $=\frac{1}{r^2}\left[x\vec{i}+y\vec{j}+z\vec{k}\right]=\frac{\vec{r}}{r^2}$
	PART-B
1.	**Prove that** $\nabla(r^n)=nr^{n-2}\,\vec{r}$. **(May/June 2003,2008)(8 M)**BTL5 **Answer : Refer Page No.2.5- Dr.M.CHANDRASEKAR**

	• $\frac{\partial r}{\partial x}=\frac{x}{r},\ \frac{\partial r}{\partial y}=\frac{y}{r},\ \frac{\partial r}{\partial z}=\frac{z}{r}$ **(2 M)** • $\nabla(r^n)\ =\vec{i}\left(nr^{n-1}\frac{\partial r}{\partial x}\right)+\vec{j}\left(nr^{n-1}\frac{\partial r}{\partial y}\right)+\vec{k}\left(nr^{n-1}\frac{\partial r}{\partial z}\right)$ **(2 M)** • $\nabla(r^n)\ =\frac{nr^{n-1}}{r}\left[x\vec{i}+y\vec{j}+z\vec{k}\right]=nr^{n-2}\vec{r}$ **(4 M)**
2.	**Prove that** $\mathrm{Curl}\,(\mathrm{Curl}\,\vec{F})=\nabla(div\,\vec{F})-\nabla^2\,\vec{F}$ **(May/June 2003,2008) (8 M)**BTL5 **Answer : Refer Page No.2.36- Dr.M.CHANDRASEKAR** • $\nabla\times(\nabla\times\vec{F})=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ \frac{\partial F_3}{\partial y}-\frac{\partial F_2}{\partial z} & \frac{\partial F_1}{\partial z}-\frac{\partial F_3}{\partial x} & \frac{\partial F_2}{\partial x}-\frac{\partial F_1}{\partial y}\end{vmatrix}$ **(3 M)** • $\nabla\times(\nabla\times\vec{F})=\sum\left\{\frac{\partial}{\partial x}(div\,\vec{F})-\nabla^2\,\vec{F}_1\right\}\vec{i}$ **(3 M)** • For proving $\mathrm{Curl}\,(\mathrm{Curl}\,\vec{F})=\nabla(div\,\vec{F})-\nabla^2\,\vec{F}$ **(2 M)**
3.	**Prove that** $\vec{F}=(y^2\cos x+z^3)\,\vec{i}+(2y\sin x-4)\,\vec{j}+3xz^2\,\vec{k}$ **is irrotational and find its scalar potential.(8 M)** BTL5 **Answer : Refer Page No.2.33- Dr.M.CHANDRASEKAR** • $\nabla\times\vec{F}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ y^2\cos x+z^3 & 2y\sin x-4 & 3xz^2\end{vmatrix}=0$ **(2 M)** $\phi_1=y^2\sin x+xz^3+f(y,z)$ • $\phi_2=y^2\sin x-4y+f(x,z)$ **(4 M)** $\phi_3=xz^3+f(x,y)$

	• $\phi = y^2 \sin x + xz^3 - 4y + c$ **(2 M)**
4.	**Prove that** $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ **is irrotational and find its scalar potential.(NOV/DEC 2015,R-13)(8 M)** BTL5 **Answer : Refer Page No.2.32- Dr.M.CHANDRASEKAR** • $\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (6xy + z^3) & (3x^2 - z) & (3xz^2 - y) \end{vmatrix} = 0$ **(2 M)** $\phi_1 = 3x^2 y + xz^3 + f(y,z)$ • $\phi_2 = 3x^2 y - yz + f(x,z)$ **(4 M)** $\phi_3 = xz^3 - yz + f(x,y)$ • $\phi = 3x^2 y + xz^3 - yz + c$ **(2 M)**
5.	**Prove that** $\vec{F} = (y^2 + 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2zx^2 - y + 2z)\vec{k}$ **is irrotational and find its scalar potential. (8 M)** BTL5 **Answer : Refer Page No.2.47- Dr.M.CHANDRASEKAR** • $\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (y^2 + 2xz^2) & (2xy - z) & (2zx^2 - y + 2z) \end{vmatrix} = 0$ **(2 M)** $\phi_1 = xy^2 + x^2 z^2 + f(y,z)$ • $\phi_2 = xy^2 - yz + f(x,z)$ **(4 M)** $\phi_3 = x^2 z^2 + xy^2 - yz + f(x,y)$ • $\phi = x^2 z^2 + xy^2 - yz + c$ **(2 M)**
6.	**Prove that** $\vec{F} = (y + z)\vec{i} + (z + x)\vec{j} + (x + y)\vec{k}$ **is irrotational and find its scalar potential. (8 M)** BTL5 **Answer : Refer Page No.2.46- Dr.M.CHANDRASEKAR** • $\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (y + z) & (z + x) & (x + y) \end{vmatrix} = 0$ **(2 M)**

	$\phi_1 = xy + xz + f(y,z)$ • $\phi_2 = xy + yz + f(x,z)$ **(4 M)** $\phi_3 = xz + yz + f(x,y)$ • $\phi = xz + xy + yz + c$ **(2 M)**
7.	**Evaluate by Green's theorem** $\int_C (xy + x^2)dx + (x^2 + y^2)dy$ **where C is the square formed by** $x = -1, x = 1, y = -1, y = 1$ **(May/June 2016 R-13) (8 M)** BTL1 **Answer : Refer Page No.2.75- Dr.M.CHANDRASEKAR** • $\int_C udx + vdy = \iint_R \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) dxdy$ $u = xy + x^2, v = x^2 + y^2 \Rightarrow \frac{\partial u}{\partial y} = x, \frac{\partial v}{\partial x} = 2x$ **(4 M)** • $\int_C (xy + x^2)dx + (x^2 + y^2)dy = \int_{-1}^{1}\int_{-1}^{1} x\,dx\,dy$ **(2 M)** • $\int_C (xy + x^2)dx + (x^2 + y^2)dy = 0$ **(2 M)**
8.	**Verify Green's theorem** $\int_C (xy + y^2)dx + (x^2)dy$ **where C is the closed curve of the region bounded by** $y = x$ and $y = x^2$ **(May/June 2013 R-13) (8 M)** BTL3 **Answer : Refer Page No.2.78- Dr.M.CHANDRASEKAR** • $\int_C udx + vdy = \iint_R \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) dxdy$ $u = xy + y^2, v = x^2 \Rightarrow \frac{\partial u}{\partial y} = x + 2y, \frac{\partial v}{\partial x} = 2x$ **(2 M)** • $\iint_R \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) dxdy = \int_0^1 \int_y^{\sqrt{y}} (x - 2y)dx\,dy = \frac{-1}{20}$ **(2 M)** • $\int_C (xy + y^2)dx + (x^2)dy = \text{Along OA} + \text{Along AO} = \int_0^1 (x^4 + 3x^3)dx + \int_1^0 (3x^2)dx$ **(2 M)**

	• $\int_C (xy+y^2)dx+(x^2)dy=\frac{19}{20}-1=\frac{-1}{20}$ **(2 M)**
9.	**Verify Green's theorem** $\int_C (x^2-xy^3)dx+(y^2-2xy)dy$ **where C is the square with vertices (0,0),(2,0),(2,2),(0,2) (May/June 2003) (8 M)** BTL3 **Answer : Refer Page No.2.80- Dr.M.CHANDRASEKAR** • $\int_C udx+vdy=\iint_R\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)dxdy$ **(2 M)** $u=x^2-xy^3, \text{v}=\text{y}^2-2\text{xy}\Rightarrow\frac{\partial u}{\partial y}=-3xy^2,\frac{\partial v}{\partial x}=-2y$ • $\iint_R\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)dxdy=\int_0^2\int_0^2(3xy^2-2y)dx\,dy=8$ **(2 M)** • $\int_C (x^2-xy^3)dx+(y^2-2xy)dy=\text{Along OA}+\text{Along AB}+\text{Along BC}+\text{Along CO}$ $=\int_0^2(x^2)dx+\int_0^2(y^2-4y)dy+\int_2^0(x^2-8x)dx+\int_2^0(y^2)dy$ **(2 M)** • $\int_C (x^2-xy^3)dx+(y^2-2xy)dy=\frac{8}{3}-\frac{16}{3}+\frac{40}{3}-\frac{8}{3}=8$ **(2 M)**
10.	**Evaluate by Green's theorem** $\int_C (y-\sin x)dx+(\cos x)dy$ **where C is the triangle OAB where** $\mathbf{O}=(\mathbf{0},\mathbf{0}),\text{A}=\left(\frac{\pi}{2},0\right),\text{B}=\left(\frac{\pi}{2},1\right)$ **(May/June 2015 R-13) (8 M)** BTL3 **Answer : Refer Page No.2.82- Dr.M.CHANDRASEKAR** • $\int_C udx+vdy=\iint_R\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)dxdy$ **(4 M)** $u=y-\sin x, \text{v}=\cos x\Rightarrow\frac{\partial u}{\partial y}=1,\frac{\partial v}{\partial x}=-\sin x$ • $\int_C (y-\sin x)dx+(\cos x)dy=\int_0^{\frac{\pi}{2}}\int_0^{\frac{2x}{\pi}}(-\sin x-1)\,dx\,dy$ **(2 M)**

	• $\int_C (y-\sin x)dx+(\cos x)dy = -\left(\frac{\pi^2+8}{4\pi}\right)$ **(2 M)**
11.	**Apply Green's theorem to evaluate** $\int_C (3x^2-8y^2)dx+(4y-6xy)dy$ **where C is the boundary of the region defined by x=0,y=0 and x+y=1 (NOV/DEC 2014 R-13) (8 M)** BTL3 **Answer : Refer Page No.2.83- Dr.M.CHANDRASEKAR** • $\int_C udx+vdy = \iint_R \left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)dxdy$ **(4 M)** $u=-8y^2+3x^2, v=4y-6xy \Rightarrow \frac{\partial u}{\partial y}=-16y, \frac{\partial v}{\partial x}=-6y$ • $\int_C (3x^2-8y^2)dx+(4y-6xy)dy = \int_0^1\int_0^{1-y} 10y\,dx\,dy$ **(2 M)** • $\int_C (3x^2-8y^2)dx+(4y-6xy)dy = \frac{5}{3}$ **(2 M)**
12.	**Verify Gauss Divergence theorem** $\vec{F}= xy^2\vec{i}+yz^2\vec{j}+zx^2\vec{k}$ over **the region bounded by** $x=0, x=1, y=0, y=2, z=0, z=3$ **(May/June 2012 R-08) (16 M)** BTL3 **Answer : Refer Page No.2.96- Dr.M.CHANDRASEKAR** • $\iint_S \vec{F}.\hat{n}\,ds = \iiint_V \nabla.\vec{F}dv$ **(2 M)** • $\nabla.\vec{F} = y^2+x^2+z^2$ **(2 M)** • $\iiint_V \nabla.\vec{F}dv = \int_0^3\int_0^2\int_0^1 (y^2+x^2+z^2)dxdydz = 28$ **(4 M)** • $\iint_S \vec{F}.\hat{n}\,ds = 8+0+18+0+2+0=28$ **(8 M)**
13.	**Verify Gauss Divergence theorem** $\vec{F}= (x^2-yz)\vec{i}+(y^2-zx)\vec{j}+(z^2-xy)\vec{k}$ over **the rectangular Parallelopiped** $0\le x\le a,\ 0\le y\le b,\ 0\le z\le c$ **(May/June 2009 R-08) (16 M)** BTL3 **Answer : Refer Page No.2.99- Dr.M.CHANDRASEKAR**

	• $\iint_S \vec{F}.\hat{n}\,ds = \iiint_V \nabla.\vec{F}dv$ **(2 M)** • $\nabla.\vec{F} = 2x+2y+2z$ **(2 M)** • $\iiint_V \nabla.\vec{F}dv = 2\int_0^c\int_0^b\int_0^a (x+y+z)dxdydz = abc(\text{a}+\text{b}+\text{c})$ **(4 M)** • $\iint_S \vec{F}.\hat{n}\,ds = \left(a^2bc - \frac{b^2c^2}{4}\right)+\left(\frac{b^2c^2}{4}\right)+\left(b^2ac-\frac{a^2c^2}{4}\right) + \left(\frac{a^2c^2}{4}\right)+\left(c^2ba-\frac{b^2a^2}{4}\right)+\left(\frac{b^2a^2}{4}\right)$ **(8 M)** $\iint_S \vec{F}.\hat{n}\,ds = abc(\text{a}+\text{b}+\text{c})$
14.	**Verify Gauss Divergence theorem for** $\vec{F} = x^3\vec{i}+y^3\vec{j}+z^3\vec{k}$ over **the cube bounded by** $x=0, x=a, y=0, y=a, z=0, z=a$ **(May/June 2014 R-13) (16 M)** BTL3 **Answer : Refer Page No.2.106- Dr.M.CHANDRASEKAR** • $\iint_S \vec{F}.\hat{n}\,ds = \iiint_V \nabla.\vec{F}dv$ **(2 M)** • $\nabla.\vec{F} = 3y^2+3x^2+3z^2$ **(2 M)** • $\iiint_V \nabla.\vec{F}dv = \int_0^a\int_0^a\int_0^a (3y^2+3x^2+3z^2)dxdydz = 3a^5$ **(4 M)** • $\iint_S \vec{F}.\hat{n}\,ds = a^5+0+a^5+0+a^5+0 = 3a^5$ **(8 M)**
15.	**Verify Gauss Divergence theorem for** $\vec{F} = 4xz\vec{i}-y^2\vec{j}+zy\vec{k}$ over **the region bounded by** $x=0, x=1, y=0, y=1, z=0, z=1$ **(May/June 2012 R-08) (16 M)** BTL3 **Answer : Refer Page No.2.109- Dr.M.CHANDRASEKAR** • $\iint_S \vec{F}.\hat{n}\,ds = \iiint_V \nabla.\vec{F}dv$ **(2 M)** • $\nabla.\vec{F} = 4z-y$ **(2 M)** • $\iiint_V \nabla.\vec{F}dv = \int_0^1\int_0^1\int_0^1 (4z-y)dxdydz = \frac{3}{2}$ **(4 M)**

	• $\iint_S \vec{F}.\hat{n}\,ds = 2+0-1+0+\frac{1}{2}+0=\frac{3}{2}$ **(8 M)**
16.	**Verify Gauss Divergence theorem for** $\vec{F}= y\vec{i}+x\vec{j}+z^2\vec{k}$ over **the cylindrical region bounded by** $x^2+y^2=9, z=0$ and $z=2$ **(Dec/Jan 2015 R-13) (16 M)** BTL3 **Answer : Refer Page No.2.103- Dr.M.CHANDRASEKAR** • $\iint_S \vec{F}.\hat{n}\,ds = \iiint_V \nabla.\vec{F}dv$ **(2 M)** • $\nabla.\vec{F}=2z$ **(2 M)** • $\iiint_V \nabla.\vec{F}dv = \int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_0^2 2z\ dxdydz = 36\pi$ **(4 M)** • $\iint_S \vec{F}.\hat{n}\,ds = 0+36\pi+0=36\pi$ **(8 M)**
17.	**Verify Stokes theorem for** $\vec{F}= (x^2+y^2)\vec{i}-2xy\vec{j}$ **taken around the rectangle bounded by** $x=\pm a, y=0, y=b$ **(May/June 2004) (16 M)** BTL3 **Answer : Refer Page No.2.122- Dr.M.CHANDRASEKAR** • $\int_C \vec{F}.\vec{dr} = \iint_S (\nabla\times\vec{F}).\ \hat{n}\,ds$ **(2 M)** • $\nabla\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (x^2+y^2) & -2xy & 0 \end{vmatrix} = -4y\vec{k}$ **(2 M)** • $\iint_S (\nabla\times\vec{F}).\ \hat{n}\,ds = \int_0^b\int_{-a}^{a}(-4y)dxdy = -4ab^2$ **(4 M)** • $\int_C \vec{F}.\vec{dr} = AB+BC+CD+DA = \left(\frac{2a^3}{3}\right)-\left(ab^2\right)-\left(2ab^2+\frac{2a^3}{3}\right)-\left(ab^2\right)=-4ab^2$ **(8 M)**
18.	**Verify Stokes theorem for** $\vec{F}= (x^2-y^2)\vec{i}+2xy\vec{j}$ **taken around the rectangle bounded by** $x=0, x=a, y=0, y=b$ **(May/June 2004) (16 M)** BTL3 **Answer : Refer Page No.2.124- Dr.M.CHANDRASEKAR**

	• $\int_C \overrightarrow{F}.\overrightarrow{dr} = \iint_S (\nabla \times \overrightarrow{F}).\ \hat{n}\, ds$ **(2 M)** • $\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (x^2 - y^2) & 2xy & 0 \end{vmatrix} = 4y\vec{k}$ **(2 M)** • $\iint_S (\nabla \times \overrightarrow{F}).\ \hat{n}\, ds = \int_0^b \int_0^a (4\,y) dxdy = 2ab^2$ **(4 M)** • $\int_C \overrightarrow{F}.\overrightarrow{dr} = OA + AB + BC + CO = \left(\frac{a^3}{3}\right) + \left(ab^2\right) + \left(ab^2 - \frac{a^3}{3}\right) + (0) = 2ab^2$ **(8 M)**
19.	**Verify Stokes theorem for** $\vec{F} = x^2\vec{i} + xy\vec{j}$ **integrated around the square in z=0 plane whose sides are along the lines** $x = 0, x = a, y = 0, y = a$ **(May/June 2008) (16 M)** BTL3 **Answer : Refer Page No.2.126- Dr.M.CHANDRASEKAR** • $\int_C \overrightarrow{F}.\overrightarrow{dr} = \iint_S (\nabla \times \overrightarrow{F}).\ \hat{n}\, ds$ **(2 M)** • $\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 & xy & 0 \end{vmatrix} = y\vec{k}$ **(2 M)** • $\iint_S (\nabla \times \overrightarrow{F}).\ \hat{n}\, ds = \int_0^a \int_0^a (\,y) dxdy = \frac{a^3}{2}$ **(4 M)** • $\int_C \overrightarrow{F}.\overrightarrow{dr} = OA + AB + BC + CO = \left(\frac{a^3}{3}\right) + \left(\frac{a^3}{2}\right) + \left(-\frac{a^3}{3}\right) = \left(\frac{a^3}{2}\right)$ **(8 M)**
20.	**Verify Stokes theorem for** $\vec{F} = (y - z + 2)\vec{i} + (yz + 4)\vec{j} - xz\,\vec{k}$ **where S is the open surface of the cube** $x = 0, x = 2, y = 0, y = 2, z = 0, z = 2$ **above the xy-plane (May/June 2005) (16 M)** BTL3 **Answer : Refer Page No.2.132- Dr.M.CHANDRASEKAR**

	• $\int_C \vec{F}.\vec{dr} = \iint_S (\nabla \times \vec{F}).\hat{n}\,ds$ **(2 M)** • $\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y-z+2 & yz+4 & -xz \end{vmatrix} = -y\vec{i} + (z-1)\vec{j} - \vec{k}$ **(2 M)** • $\iint_S (\nabla \times \vec{F}).\hat{n}\,ds = (-4)+(4)+(4)+(-4)+(-4) = -4$ **(4 M)** • $\int_C \vec{F}.\vec{dr} = OA + AC + CB + BO = (4)+(8)+(-8)+(-8) = (-4)$ **(8 M)**
21.	**Using Stokes theorem to Evaluate** $\int_C \vec{F}.\vec{dr}$ **where** $\vec{F} = (y^2)\vec{i} + (x^2)\vec{j} - (x+z)\vec{k}$ **and C is the boundary of the triangle with vertices (0,0,0), (1,0,0) and (1,1,0)** **(8 M)** BTL3 **Answer : Refer Page No.2.137- Dr.M.CHANDRASEKAR** • $\int_C \vec{F}.\vec{dr} = \iint_S (\nabla \times \vec{F}).\hat{n}\,ds$ **(2 M)** • $\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y^2 & x^2 & -(x+z) \end{vmatrix} = \vec{j} + 2(x-y)\vec{k}$ **(2 M)** • $\iint_S (\nabla \times \vec{F}).\hat{n}\,ds = \int_0^1\int_0^x 2(\mathrm{x}-\mathrm{y})\,dydx = \frac{1}{3}$ **(4 M)**
	UNIT-III ANALYTIC FUNCTIONS
	Analytic functions – Necessary and sufficient conditions for analyticity in Cartesian and polar coordinates – Properties – Harmonic conjugates – Construction of analytic function – Conformal mapping – Mapping by $w = z + c, cz, \frac{1}{z}, z^2$ – Bilinear transformation
	PART-A
1.	**Show that the function** $f(z) = \bar{z}$ **is no where differentiable. (DEC/JAN-2013) (NOV/DEC-2015)** BTL2

	Given $w = f(z) = \bar{z}$ $\therefore u + iv = x - iy \Rightarrow u = x, v = -y$ $u_x = 1, v_x = 0$ $u_y = 1, v_y = -1$ $\therefore u_x \neq v_y$ So C-R equations are not satisfied for any x and y. $\therefore f(z)$ is not differentiable anywhere. Hence not analytic anywhere.
2	**Test the analyticity of the function** $w = \sin z$ BTL4 Given $w = \sin z$ $u + iy = \sin(x + iy)$ $= \sin x \cos iy + \cos x \sin(iy)$ $= \sin x \cosh y + i \cos x \sinh y$ $\Rightarrow u = \sin x \cosh y \; ; \quad v = \cos x \sinh y$ $\therefore u_x = \cos x \cosh y ; \quad v_x = -\sin x \sinh y$ $u_y = \sin x \sinh y \; ; \quad v_y = \cos x \cosh y$ $\therefore u_x = v_y, \quad u_y = -v_x$ So C-R equations are satisfied for all any x and y and $u_x, ,u_y, v_x, v_y$ are continuous $\therefore f(z)$ is analytic everywhere.
3	**Find the constants a,b,c if** $f(z) = x + ay + i(bx + cy)$ **is analytic. (DEC/JAN-2014)** BTL1 Let $u + iv = f(z)$ Since $f(z)$ is analytic, u and v satisfy the C-R Equations. $u_x = v_y, \quad u_y = -v_x$ $here\, u = x + ay, v = bx + cy$ $u_x = 1, \; v_x = b$ $u_y = a, \; v_y = c$ $\therefore u_x = v_y \Rightarrow c = 1;$ $u_y = -v_x \Rightarrow a = -b$
4	**Show that** $u = 2x - x^3 + 3xy^2$ **is harmonic** BTL2

	Given $u = 2x - x^3 + 3xy^2$ $u_x = 2 - 3x^2 + 3y^2; u_y = 6xy$ $u_{xx} = -6x; \quad u_{yy} = 6x$ $\therefore u_{xx} + u_{yy} = -6x + 6x = 0.$ Therefore u is harmonic
5	**Show that the function** $u = y + e^x \cos y$ **is harmonic** BTL2 Given $u = y + e^x \cos y$ $u_x = e^x \cos y, \quad u_y = 1 + e^x(-\sin y)$ $u_{xx} = e^x \cos y, \quad u_{yy} = -e^x \cos y$ $u_{xx} + u_{yy} = e^x \cos y - e^x \cos y = 0$ Therefore u is harmonic
6	**Show that** $x^2 + iy^3$ **is not analytic anywhere.** BTL2 Let $u + iv = x^2 + iy^3$ $\therefore u = x^2, \quad v = y^3$ $u_x = 2x, \quad v_x = 0$ $u_y = 0, \quad v_y = 3y^2$ $\therefore u_x \neq v_y, \quad u_y = -v_x$ $\therefore$ The function is not analytic. But, when $x = 0, y = 0$ the C-R Equations are satisfied.
7	**For the conformal mapping** $f(z) = z^2$, **find the scale factor at** $z = i$ BTL1 Given $f(z) = z^2,$ $\therefore f'(z) = 2z$ Scale factor at $z = i$ is $\|f'(i)\| = \|2i\| = 2$
8	**Find the image of x** $= 2$ **under the transformation** $w = \frac{1}{z}$ BTL1

	$\text{Given } w=\frac{1}{z}\Rightarrow z=\frac{1}{w}=\frac{\overline{w}}{w\overline{w}}$ $\Rightarrow x+iy=\frac{u-iv}{u^2+v^2}$ $\therefore x=\frac{u}{u^2+v^2}$ $\therefore$ The image of $x=2$ *is* $\frac{u}{u^2+v^2}=2\Rightarrow u^2+v^2-\frac{u}{2}=0$ which is a circle in the w – plane.
9	**Find the image of** $x=k$ **under the transformation** $w=\frac{1}{z}$**.** BTL1 $\text{Given } w=\frac{1}{z}\Rightarrow z=\frac{1}{w}=\frac{\overline{w}}{w\overline{w}}$ $\Rightarrow x+iy=\frac{u-iv}{u^2+v^2}$ $\therefore x=\frac{u}{u^2+v^2}$ $\therefore$ The image of $x=k$ is $\frac{u}{u^2+v^2}=k\Rightarrow u^2+v^2-\frac{u}{k}=0$ which is a circle in the w – plane
10	**Find the image of the circle** $\lvert z\rvert=2$ **under the transformation** $w=3z$ **.(NOV/DEC-2014)** BTL1 Given $w=3z$ $\lvert w\rvert=3\lvert z\rvert$ $=3\times 2$ $=6$ $\therefore$ The image of the circle $\lvert z\rvert=2$ is the circle $\lvert w\rvert=6$ in the w-plane. $\therefore \sqrt{u^2+v^2}=6,$ $\Rightarrow u^2+v^2=36,$ which is a circle
11	**Find the image of the circle** $\lvert z\rvert=2$ **under the transformation** $w=z+3+2i$**.** BTL1 Given $w=z+3+2i$

	$u+iv = x+iy+3+2i$ $\therefore\ u = x+3 \Rightarrow\ x = u-3$ $v = y+2 \Rightarrow\ y = v-2$ $\lvert z \rvert = 2 \Rightarrow \sqrt{x^2+y^2} = 2$ $\Rightarrow\ x^2+y^2 = 4$ $\Rightarrow (u-3)^2+(v-2)^2 = 4$
12	**Find the image of the line** $x-y+1=0$ **under the map** $w=\frac{1}{z}$**.** BTL1 Given $w=\frac{1}{z} \Rightarrow z=\frac{1}{w}=\frac{\overline{w}}{w\overline{w}}$ $\Rightarrow\ x+iy=\frac{u-iv}{u^2+v^2}$ $\therefore x=\frac{u}{u^2+v^2}, y=\frac{-v}{u^2+v^2}$ The image of the line $x-y+1=0$ is $\frac{u}{u^2+v^2}+\frac{v}{u^2+v^2}+1=0$ $\Rightarrow u^2+v^2+u+v=0$ which is a circle in the $w-$plane
13	**Find the fixed points of the transformation** $w=\frac{6z-9}{z}$**.** BTL1 The given transformation $w=\frac{6z-9}{z}$. The fixed points are given points by $w = z$ $\Rightarrow\ z=\frac{6z-9}{z}$ $\Rightarrow\ z^2=6z-9$ $\Rightarrow\ z^2-6z-9=0$ $\Rightarrow\ (z-3)^2=0$ $\Rightarrow\ z=3,3$
14	**Find the fixed points of the mapping** $w=\frac{3-z}{1+z}$**.** BTL1 The given maps $w=\frac{3-z}{1+z}$

	The fixed points are given by $w = z$ $\therefore \quad z = \frac{3-z}{1+z} \Rightarrow z + z^2 = 3 - z$ $\Rightarrow z + z^2 - 3 + z = 0$ $\Rightarrow z^2 + 2z - 3 = 0$ $\Rightarrow (z+3)(z-1) = 0$ $\Rightarrow z = -3, 1$
15	**Find the fixed points of the mapping** $w = \frac{2z+6}{z+7}$. **(DEC/JAN-2015)** BTL1 The given map is $w = \frac{2z+6}{z+7}$. The fixed points are given by $w = z$ $\therefore \quad z = \frac{2z+6}{z+7} \Rightarrow 7z + z^2 = 2z + 6$ $\Rightarrow 7z + z^2 - 2z + 6 = 0$ $\Rightarrow z^2 + 5z - 6 = 0$ $\Rightarrow (z+6)(z-1) = 0$ $\Rightarrow z = 1, -6$
16	**Find the bilinear map which maps points** $\infty, i, 0$ **of the z plane onto** $0, i, \infty$ **of the w-plane.** BTL1 Given $z_1 = \infty, z_2 = i, z_3 = 0$ which are mapped onto $w_1 = 0, w_2 = i, w_3 = \infty$ Since $z_1 = \infty$ & $w_3 = \infty$, omitting the factors involving z_1 & w_3 The Bilinear map is, $\frac{w - w_1}{w_2 - w_1} = \frac{z_2 - z_3}{z - z_3}$ $\frac{w-0}{i-0} = \frac{i-0}{z}$ $\Rightarrow w = -\frac{1}{z}$
17	**Define the Conformal Mapping**. BTL1 Definition: A transformation that preserves angles between every pair of curves through a Point, both in magnitude and sense, is said to be conformal at that point.
18	**State sufficient condition for analytic function. (DEC/JAN-2016)** BTL1

	If the partial derivatives u_x, u_y, v_x, and v_y are all continuous in D and $u_x = v_y,\ u_y = -v_x$. Then the function *f(z)* is analytic in a domain D.
19	**Find the constants a, b if $f(z) = x + 2ay + i(3x + by)$ is analytic.** BTL1 Given $f(z) = x + 2ay + i(3x + by)$ is analytic. $\Rightarrow u_x = v_y,\ u_y = -v_x$ ....(1) Here $u = x + 2ay$ and $v = 3x + by$ Thus (1) gives $1 = b$ and $2a = -3$ $\Rightarrow a = -\frac{3}{2}$ and $b = -1$
20	**State the Cauchy Riemann equations in polar coordinates satisfied by an analytic Function.** BTL1 Cauchy Riemann equations in polar coordinates are given by $u_r = \frac{1}{r} v_\theta$ and $v_r = -\frac{1}{r} u_\theta$ where u and v are functions of r *and* θ.
21	**Find the critical points of the transformation $w = 1 + \frac{2}{z}$. (NOV/DEC-2016)** BTL1 The critical points of the transformation are obtained by $f'(z) = 2z$ $Hence\ -\frac{2}{z^2} = 0$ $\Rightarrow -\frac{2}{0} = z^2$ $\Rightarrow z = \infty$ is the critical point of the given transformation.
22	**Find the image of the region $x > c$, where $c > 0$ under the transformation $w = \frac{1}{z}$.** BTL1 $w = \frac{1}{z}. \Rightarrow z = \frac{1}{w}$ Let $z = x + iy$ and $w = u + iv$ $x + iy = \frac{1}{u + iv} = \frac{u - iv}{(u + iv)(u - iv)} = \frac{u - iv}{u^2 + v^2}$ $\therefore x = \frac{u}{u^2 + v^2}$ *and* $y = \frac{-v}{u^2 + v^2}$ $\boldsymbol{x} > c \Rightarrow x = \frac{u}{u^2 + v^2} > c$ $u > cu^2 + cv^2$

	$\mathrm{u}^2 + \mathrm{v}^2 < \frac{u}{c}$ $\mathrm{u}^2 + \mathrm{v}^2 - \frac{u}{c} < 0.$ This refers to the inside of the circle center $(\frac{1}{2c}, 0)$ and radius $\frac{1}{2c}$.
23	**Show that an analytic function with constant real part is constant.** BTL2 Let $f(z) = u + iv$ be analytic. $\Rightarrow u_x = v_y \; and \; u_y = -v_x$ Given that $u = constant. = c(say). \Rightarrow u_x = 0 \;\; and \;\; v_y = 0$ $\Rightarrow u_y = 0 \;\; and \;\; -v_x = 0$ ⇒ v is independent of *x* and *y*. $\Rightarrow v$ is constant $\Rightarrow f(z) = u + iv = c + ic$ is a constant.
24	**Find the critical points of the transformation $w^2 = (z-\alpha)(z-\beta)$. (DEC/JAN-2010) (NOV/DEC-2016)** BTL1 Let $w^2 = (z-\alpha)(z-\beta)$. Then, $2w\frac{dw}{dz} = (z-\alpha).1 + (z-\beta).1$ The Critical points of $w = f(z)$ is given by, $\frac{dw}{dz} = 0 \Rightarrow (z-\alpha).1 + (z-\beta).1 = 0 \quad \Rightarrow z = \frac{\alpha+\beta}{2}$. Also, $\frac{dz}{dw} = 0 \Rightarrow \frac{2w}{(z-\alpha)+(z-\beta)} = 0. \Rightarrow w = 0, (z-\alpha) + (z-\beta) = 0 \quad \Rightarrow z = \alpha, \beta.$ The critical points are $z = \alpha, \beta, \frac{\alpha+\beta}{2}$.
25	**Write cross ratio of four points. (NOV/DEC-2018)** BTL1 The cross ratio of four points. $\frac{(w_1-w_2)(w_3-w_4)}{(w_2-w_3)(w_4-w_1)} = \frac{(z_1-z_2)(z_3-z_4)}{(z_2-z_3)(z_4-z_1)}$ is invariant under the bilinear transformation
26	**Verify f(z)= z^3 is analytic or not.** BTL3 Let $f(z) = u + iv = z^3 = (x+iy)^3$ $u + iv = (x^3 - 3xy^2) + i(3x^2y - y^3)$ $u = (x^3 - 3xy^2)$ and $v = (3x^2y - y^3)$ $u_x = (3x^2 - 3y^2) \; and \; u_y = -6xy$ $v_x = 6xy \;\; and \;\; v_y = (3x^2 - 3y^2)$ $u_x = v_y$ and $u_y = -v_x$. Hence the C-R Equations are satisfied. Therefore f(z)= z^3 is analytic
27	**If $f(z) = u + iv$ is an analytic function ,prove that u is a harmonic function.** BTL5 $f(z) = u + iv$ be analytic. $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}; \quad \frac{\partial u}{\partial y} = \frac{-\partial v}{\partial x}$ … … … … … (1) Now, $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial x}\left(\frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{-\partial v}{\partial x}\right)$ (since by (1))

	$= \frac{\partial^2 v}{\partial x \partial y} - \frac{\partial^2 v}{\partial y \partial x} = 0$ $\therefore u$ is harmonic
	PART-B
1.	**If** $f(z)$ **is an analytic function, Prove that** $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\|f(z)\|^2 = 4\|f'(z)\|^2$ **(NOV/DEC 2014) (8 M)** BTL5 **Answer : Refer Page No.3.31- Dr.M.CHANDRASEKAR** • C-R Equations are $u_x = v_y$, $u_y = -v_x$ **(2 M)** • $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\|f(z)\|^2 = 2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]$ **(4 M)** • $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\|f(z)\|^2 = 4\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2\right] = 4\|f'(z)\|^2$ **(2 M)**
2.	**If** $f(z)=u+iv$ **is analytic, Prove that** $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log\|f(z)\| = 0$ **(MAY/JUNE 2002) (8 M)** BTL5 **Answer : Refer Page No.3.33- Dr.M.CHANDRASEKAR** • C-R Equations are $u_x = v_y$, $u_y = -v_x$ **(2 M)** • $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log\|f(z)\| = \frac{(u^2+v^2)[u_x^2+v_x^2+u_y^2+v_y^2+u(u_{xx}+u_{yy}) + v(v_{xx}+v_{yy}) - 2[(uu_x+vv_x)^2+(uu_y+vv_y)^2]}{(u^2+v^2)^2}$ **(4 M)** • Since the function f(z) is analytic, it satisfies C-R equations and hence the function is harmonic. **(2 M)** $\therefore \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log\|f(z)\| = 0$
3.	**Prove that** $u = x^2 - y^2, v = \frac{-y}{x^2+y^2}$ are harmonic but $u+iv$ **is not regular function.** **(NOV/DEC 2013) (8 M)** BTL5 **Answer : Refer Page No.3.44- Dr.M.CHANDRASEKAR**

	• For Proving u is harmonic $u_{xx}+u_{yy}=2-2=0$ **(2 M)** • For Proving v is harmonic $v_{xx}+v_{yy}=\left(\frac{2y^3-6x^2y}{(x^2+y^2)^3}\right)+\left(-\frac{(2y^3-6x^2y)}{(x^2+y^2)^3}\right)=0$ **(2 M)** • But $u_x\neq v_y$, $u_y\neq -v_x \Rightarrow f(z)=u+iv$ is not a regular function **(2 M)**
4.	**In a two dimensional flow, the stream function is $\psi=\tan^{-1}\left(\frac{y}{x}\right)$ Find the velocity Potential ϕ . (NOV/DEC 2016) (8 M)** BTL1 **Answer : Refer Page No.3.50- Dr.M.CHANDRASEKAR** • $\frac{\partial\psi}{\partial x}=\frac{-y}{x^2+y^2}$; $\frac{\partial\psi}{\partial y}=\frac{x}{x^2+y^2}$ **(2 M)** • $\phi=\int\left(\frac{\partial\psi}{\partial y}dx-\frac{\partial\psi}{\partial x}dy\right)$ **(2 M)** • $\phi=\log(x^2+y^2)+c$ **(4 M)**
5.	**Show that the function $u=\frac{1}{2}\log(x^2+y^2)$ is harmonic and find its harmonic conjugate (MAY/JUNE 2016) (8 M)** BTL2 **Answer : Refer Page No.3.52- Dr.M.CHANDRASEKAR** • $\frac{\partial u}{\partial x}=\frac{x}{x^2+y^2}$; $\frac{\partial u}{\partial y}=\frac{y}{x^2+y^2}$ **(2 M)** • For Proving u is harmonic $u_{xx}+u_{yy}=\left(\frac{y^2-x^2}{(x^2+y^2)^2}\right)+\left(-\frac{y^2-x^2}{(x^2+y^2)^2}\right)=0$ **(2 M)** • $v=\tan^{-1}\left(\frac{y}{x}\right)+c$ **(4 M)**
6.	**Prove that $e^x[x\cos y-y\sin y]$ can be the real part of an analytic function and determine its harmonic conjugate (NOV/DEC 2013) (8 M)** BTL5 **Answer : Refer Page No.3.55- Dr.M.CHANDRASEKAR**

	• $\frac{\partial u}{\partial x} = e^x x\cos y + e^x \cos y - e^x y\sin y$ $\frac{\partial u}{\partial y} = -e^x x\sin y - e^x y\cos y - e^x \sin y$ **(2 M)** • For Proving u is harmonic $u_{xx} + u_{yy} = \left(e^x x\cos y + 2e^x \cos y - e^x y\sin y\right) + \left(-e^x x\cos y - 2e^x \cos y + e^x y\sin y\right) = 0$ **(2 M)** • $v = e^x x\sin y + e^x y\cos y + c$ **(4 M)**
7.	**Find an analytic function** $f(z) = u + iv$ **whose real part is** $e^x[x\cos y - y\sin y]$ **(8 M)** BTL1 **Answer : Refer Page No.3.64- Dr.M.CHANDRASEKAR** • $\frac{\partial u}{\partial x} = e^x x\cos y + e^x \cos y - e^x y\sin y$ $\frac{\partial u}{\partial y} = -e^x x\sin y - e^x y\cos y - e^x \sin y$ **(2 M)** • $\frac{\partial u}{\partial x}(z,0) = e^z + ze^z$ $\frac{\partial u}{\partial y}(z,0) = 0$ **(2 M)** $f(z) = ze^z + c$ **(4 M)**
8.	**Find an analytic function** $f(z) = u + iv$ **whose real part is** $e^{2x}[x\cos 2y - y\sin 2y]$ **(8 M)** BTL1 **Answer : Refer Page No.3.66- Dr.M.CHANDRASEKAR** • $\frac{\partial u}{\partial x} = 2e^{2x} x\cos 2y + e^{2x} \cos 2y - 2e^{2x} y\sin 2y$ $\frac{\partial u}{\partial y} = -2e^{2x} x\sin 2y - 2e^{2x} y\cos 2y - e^{2x} \sin 2y$ **(2 M)** • $\frac{\partial u}{\partial x}(z,0) = e^{2z} + 2ze^{2z}$ $\frac{\partial u}{\partial y}(z,0) = 0$ **(2 M)**

	• $f(z) = ze^{2z} + c$ **(4 M)-**
9.	**Prove that the function** $v = e^{-x}[x\cos y + y\sin y]$ **is harmonic and determine the corresponding analytic function** $f(z) = u + iv$ **(8 M)** BTL5 **Answer : Refer Page No.3.69- Dr.M.CHANDRASEKAR** • $\frac{\partial v}{\partial x} = -e^{-x}x\cos y + e^{-x}\cos y - e^{-x}y\sin y$ $\frac{\partial v}{\partial y} = -e^{-x}x\sin y + e^{-x}\,y\cos y + e^{-x}\sin y$ **(2 M)** • For Proving u is harmonic $v_{xx} + v_{yy} = \left(e^{-x}[(x-2)\cos y + y\sin y]\right) + \left(e^{-x}[(2-x)\cos y - y\sin y]\right) = 0$ **(2 M)** • $\frac{\partial v}{\partial x}(z,0) = e^{-z}(1-z)$ $\frac{\partial v}{\partial y}(z,0) = 0$ **(2 M)** • $f(z) = ize^{-z} + c$ **(2 M)**
10.	**Given that** $u = \frac{\sin 2x}{\cosh 2y - \cos 2x}$ **find the analytic function whose real part is u.** **(NOV/DEC 2014)(MAY/JUNE 2006) (8 M)** BTL1 **Answer : Refer Page No.3.71- Dr.M.CHANDRASEKAR** • $\frac{\partial u}{\partial x}(z,0) = -\text{cosec}^2 z$ $\frac{\partial u}{\partial y}(z,0) = 0$ **(4 M)-** • $f(z) = \cot z + c$ **(4 M)**
11.	**If** $f(z) = u + iv$ **is analytic, find** $f(z)$ **given that** $u + v = \frac{\sin 2x}{\cosh 2y - \cos 2x}$ **(NOV/DEC 2015) (8 M)** BTL1 **Answer : Refer Page No.3.74- Dr.M.CHANDRASEKAR**

	• $\frac{\partial V}{\partial x}(z,0) = -\text{cosec}^2 z$ $\frac{\partial V}{\partial y}(z,0) = 0$ **(4 M)** • $f(z) = \left(\frac{1+i}{2}\right)\cot z + c$ **(4 M)**
12.	**Find the image of** $\lvert z-3\rvert = 3$ **under the mapping** $w = \frac{1}{z}$ **(NOV/DEC 2010) (8 M)** BTL1 **Answer : Refer Page No.3.108- Dr.M.CHANDRASEKAR** • $x = \frac{u}{u^2+v^2}$ & $y = \frac{-v}{u^2+v^2}$ **(4 M)** • The image of the circle $\lvert z-3\rvert = 3$ is the straight line $u = \frac{1}{6}$ **(4 M)**
13.	**Find the image of** $\lvert z+i\rvert = 1$ **under the mapping** $w = \frac{1}{z}$ **(NOV/DEC 2013) (8 M)** BTL1 **Answer : Refer Page No.3.109- Dr.M.CHANDRASEKAR** • $x = \frac{u}{u^2+v^2}$ & $y = \frac{-v}{u^2+v^2}$ **(4 M)** • The image of the circle $\lvert z+i\rvert = 1$ is the straight line $v = \frac{1}{2}$ **(4 M)**
14.	**Find the image of** $1 < y < 2$ **under the mapping** $w = \frac{1}{z}$ **(MAY/JUNE 2014) (8 M)** BTL1 **Answer : Refer Page No.3.110- Dr.M.CHANDRASEKAR** • $x = \frac{u}{u^2+v^2}$ & $y = \frac{-v}{u^2+v^2}$ **(4 M)** • $1 < y < 2$ is mapped onto the region between the circles $u^2+v^2+v=0$ and $2(u^2+v^2)+v=0$ **(4 M)**
15.	**Find the image of** $\lvert z-2i\rvert = 1$ **under the mapping** $w = \frac{1}{z}$ **(NOV/DEC 2007) (8 M)** BTL1

	Answer : Refer Page No.3.112- Dr.M.CHANDRASEKAR •
16.	**Find the bilinear transformation which maps $-1,-i,1$ in the z-plane $\infty,i,0$ in the w-plane respectively. (8 M)** BTL1 **Answer : Refer Page No.3.132- Dr.M.CHANDRASEKAR** • $\frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)}=\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$ **(2 M)** • $w=\frac{(1-z)}{(1+z)}$ **(6 M)**
17.	**Find the bilinear transformation which maps $\infty,i,0$ onto $0,i,\infty$ respectively. (8 M)** BTL1 **Answer : Refer Page No.3.133- Dr.M.CHANDRASEKAR** • $\frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)}=\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$ **(2 M)** • $w=\frac{-1}{z}$ **(6 M)**
18.	**Find the bilinear transformation which maps $z=1,0,-1$ onto $w=\infty,-1,0$ respectively. (8 M)** BTL1 **Answer : Refer Page No.3.133- Dr.M.CHANDRASEKAR** • $\frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)}=\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$ **(2 M)** • $w=\frac{z+1}{z-1}$ **(6 M)**
19.	**Find the bilinear transformation which maps $-1,0,1$ onto $-1,-i,0$ respectively. Show that under this transformation the upper half of the z-plane maps onto the interior of the unit circle $\|w\|=1$ (8 M)** BTL1 **Answer : Refer Page No.3.134- Dr.M.CHANDRASEKAR** • $\frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)}=\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$ **(2 M)**

	• $w = \dfrac{1-iz}{z-i}$ **(2 M)** • $x = \dfrac{2u}{u^2+(v-1)^2}$ & $y = \dfrac{-(u^2+v^2-1)}{u^2+(v-1)^2}$ **(2 M)** • For proving the upper half of the z-plane maps onto the interior of the unit circle $\lvert w\rvert \le 1$ **(2 M)**
	UNIT IV- COMPLEX INTEGRATION
	Line integral – Cauchy's integral theorem – Cauchy's integral formula – Taylor's and Laurent's series – Singularities – Residues – Residue theorem – Application of residue theorem for evaluation of real integrals – Use of circular contour and semicircular contour.
Q.No.	**PART-A**
1	**State Cauchy integral theorem. (NOV/DEC 2014)(MAY/JUNE 2016)** BTL1 If a function $f(z)$ is analytic and its derivative $f'(z)$ is continuous at all points inside and on a simple closed curve C, then $\int_C f(z)dz = 0$.
2	**State Cauchy integral formula.** BTL1 If $f(z)$ is analytic inside and on a simple closed curve C in the region R and if 'a' is any point in R then $\int_C \dfrac{f(z)}{z-a}dz = 2\pi i f(a)$ where the integration around C taken in the positive direction.
3	**State Cauchy integral formula for derivatives. (NOV/DEC 2010)** BTL1 If a function f(z) is analytic within and on a simple closed curve c and 'a' is any point lying in it, then $\int_C \dfrac{f(z)}{(z-a)^{n+1}}dz = \begin{cases} \dfrac{2\pi i}{n!} f^n(a) & ;\ \text{a lies inside c} \\ 0 & ;\text{a lies outside c} \end{cases}$
4	**State Cauchy Residue Theorem (NOV/DEC 2012)** BTL1 If f (z) is analytic at all points inside and on a simple closed curve C except at a Finite number of points $z_1, z_2, z_3, \ldots\ldots, z_n$ inside C then $\int_C f(z)dz = 2\pi i\left[\text{sum of residues of } f(z)\right]$

5	**Evaluate** $\int_C \frac{dz}{z-2}$ **where C is the square with vertices (0,0), (1,0), (1,1), (0,1).** BTL5 Given C is the square with vertices (0,0), (1,0), (1,1), (0,1). ie) x=1,y=1. Since $\int_C \frac{dz}{z-2}$. Equating the denominator to zero. $z-2=0, \Rightarrow z=2$. Which lies outside C.
6	**Evaluate** $\int_C \frac{3z^2+7z+1}{z-3}dz$ **where C is** $\|z\|=2$ BTL5 Given $\|z\|=2$ that is, $x^2+y^2=2^2$ with center (0,0) and radius 2. Given $\int_C \frac{3z^2+7z+1}{z-3}dz$. Equating the denominator to zero. $(z-3)^2=0 \Rightarrow z=3$ which lies outside C. $\therefore$ By Cauchy's integral formula $\int_C \frac{3z^2+7z+1}{z-3}dz=0$.
7	**Evaluate** $\int_C \frac{\cos \pi z}{z-1}dz$ **where C is** $\|z\|=2$ BTL5 Given $\|z\|=2$ that is, $x^2+y^2=2^2$ with center (0,0) and radius 2. Given $\int_C \frac{\cos \pi z}{z-1}dz$. Equating the denominator to zero. $z-1=0, \Rightarrow z=1$. Which lies inside C. $\therefore$ By Cauchy's integral formula $\int_C \frac{dz}{z-a}=2\pi i\, f(a)$. *Here* $a=1, f(z)=\cos \pi z \Rightarrow f(a)=f(1)=\cos \pi=-1$. $\therefore \int_C \frac{\cos \pi z}{z-1}dz=2\pi i(-1)=-2\pi i$.
8	**Evaluate** $\int_C \tan z\, dz$ **where C is** $\|z\|=2$ **(NOV/DEC 2015)** BTL5 Given $\|z\|=2$ that is, $x^2+y^2=2^2$ with center (0,0) and radius 2. Given $\int_C \tan z\, dz=\int_C \frac{\sin z}{\cos z}dz$. Equating the denominator to zero. Cos z = 0 = $\cos\frac{\pi}{2} \Rightarrow z=\frac{\pi}{2}$ =1.732 . Which lies inside C.

	$\therefore$ By Cauchy's integral formula $\int_C \frac{dz}{z-a} = 2\pi i\, f(a)$. $Here\ \ a = \frac{\pi}{2},\ f(z) = \sin z \Rightarrow f(a) = f(\frac{\pi}{2}) = \sin\frac{\pi}{2} = 1$. $\therefore \int_C \tan z\, dz = 2\pi i(1) = 2\pi i$
9	**Evaluate the integral** $\int_C (z^2+2z)\,dz$ **where C is** $\lvert z\rvert = 1$ BTL5 Given $\lvert z\rvert = 1$. that is, $x^2+y^2=1$ with centre (0,0)and radius 1. $f(z) = z^2+2z$ is a function which is analytic in the region bounded by C Hence by Cauchy's theorem $\int_C (z^2+2z)\,dz = 0$.
10	**Find the contour C:** $\lvert z\rvert < 1$ **for which** $\int_C \frac{e^z}{(z+1)^2 (z+1)} dz$ **=0.** BTL1 $\int_C \frac{e^z}{(z+1)^2 (z+1)} dz$ **=0** when $\lvert z\rvert < 1$. [since the points lies outside the contour, then the integral value is 0.]
11	**Evaluate** $\int_C \frac{dz}{(z-3)^2}$ **where C is** $\lvert z\rvert = 1$ BTL5 Given $\lvert z\rvert = 1$. that is,) $x^2+y^2=1$ with center (0,0) and radius 1. $\int_C \frac{dz}{(z-3)^2}$. Equating the denominator to zero. $(z-3)^2 = 0 \Rightarrow z = 3$ which lies outside C. $\therefore$ By Cauchy's integral formula for derivatives $\int_C \frac{dz}{(z-3)^2} = 0$.
12	**Evaluate** $\int_c \frac{e^z dz}{z-2}$ **, where C is the unit circle with centre as origin.** BTL5 **(MAY/JUNE 2009)** $f(z) = \frac{e^z}{z-2}$ z=2 lies outside C. $f(z)$ is analytic inside and on C.

	$f'(z)$ is continuous in C, By Cauchy's integral theorem $\int_c f(z)dz = 0$
13	**Define Taylor's series.** BTL1 Definition: If $f(z)$ is analytic inside a circle C with its centre at z = a then, For all z inside c, $f(z) = f(a) + \frac{f'(a)}{1!}(z-a) + \frac{f''(a)}{2!}(z-a)^2 + ... + \frac{f^n(a)}{n!}(z-a)^n + ... + \infty$.
14	**Define Laurent's series.** BTL1 Definition: If C_1 and C_2 are two concentric circles with centre "a" and radii r_1 and r_2 ($r_1 < r_2$) and if $f(z)$ is analytic on C_1 and C_2 and in the annulus region between them, then at any point z in R $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n + \sum_{n=1}^{\infty} \frac{b_n}{(z-a)^n}$, where $a_n = \frac{1}{2\pi i} \int_{C_1} \frac{f(z)}{(z-a)^{n+1}} dz$ and $b_n = \frac{1}{2\pi i} \int_{C_2} \frac{f(z)}{(z-a)^{1-n}} dz$ The integrals being taken in the anticlockwise direction.
15	**Define Essential singularity** BTL1 Definition: A singular point z =a is called an essential singular point of $f(z)$ if the Laurent's series of $f(z)$ containing negative powers of z.
16	**Discuss the nature of singularities** $f(z) = e^{\frac{1}{z}}$ **.(NOV/DEC 2015)(MAY/JUNE 2012)** BTL6 $f(z) = e^{\frac{1}{z}} = 1 + \frac{\left(\frac{1}{z}\right)}{1!} + \frac{\left(\frac{1}{z}\right)^2}{2!} + \frac{\left(\frac{1}{z}\right)^3}{3!} + ...$ $= 1 + z^{-1} + \frac{z^{-2}}{2!} + \frac{z^{-3}}{3!} + ...$ Therefore z =0 is an essential singularity, since the principal part contains negative powers of z.
17	**Define removable singularity** BTL1 Definition: A singular point z=a is called a removable singular point of $f(z)$, if the Laurent's series of $f(z)$ containing positive powers of z.
18	**Find the nature of the singularity** $f(z) = \frac{\sin z}{z}$. BTL1

	$f(z)=\frac{\sin z}{z}=\frac{1}{z}\left(z-\frac{z^3}{3!}+\frac{z^5}{5!}+......\right)=1-\frac{z^2}{3!}+\frac{z^4}{5!}$ -.... There is no negative power of z. Therefore $z=0$ is a removable singularity.
19	**Define isolated singularity with an example** BTL1 Definition: A point $z=z_0$ is said to be isolated singularity of $f(z)$ i) If $f(z)$ is not analytic at $z=z_0$ ii)There exist neighborhoods of $z=z_0$ containing no other singularity Example: $f(z)=\frac{1}{(z-1)(z-2)}$ has two isolated singularity namely z =1 and z =2.
20	**Find the singularities of** $f(z)=\frac{z^2+4}{z^2+2z+2}$**.** BTL1 Given $f(z)=\frac{z^2+4}{z^2+2z+2}$**.** [The singularities are poles] The poles of $f(z)$ are given by equating the denominator to zero. $z^2+2z+2=0, \quad z=\frac{-2\pm\sqrt{4-8}}{2}=-1\pm i$. Which is a pole of order 1.
21	**Find the singularities of the function** $f(z)=\frac{\cot\pi z}{(z-a)^3}$ BTL1 Given $f(z)=\frac{\cot\pi z}{(z-a)^3}=\frac{\cos\pi z}{\sin\pi z(z-a)^3}$ $i.e. \sin\pi z(z-a)^3=0 \quad\Rightarrow\quad \sin\pi z=0 \quad (or)(z-a)^3=0$ Now $(z-a)^3=0$ $z=a$ is a pole of order 3 and then $\sin\pi z=0$ $\pi z=n\pi\Rightarrow \quad z=\pm n, \quad n=0,1,2,3.......$ $z=\pm n$ are simple poles.
22	**State nature of the singularities of** $f(z)=\sin\left(\frac{1}{z+1}\right)$ BTL1 Given $f(z)=\sin\left(\frac{1}{z+1}\right)$ $\sin\left(\frac{1}{z+1}\right)=\left(\frac{1}{z+1}\right)-\frac{\left(\frac{1}{z+1}\right)^3}{3!}+\frac{\left(\frac{1}{z+1}\right)^5}{5!}+.......$

	$=\left(\frac{1}{z+1}\right)-\frac{1}{3!}\left(\frac{1}{z+1}\right)^3+\frac{1}{5!}\left(\frac{1}{z+1}\right)^5-........$ Z=-1 is an essential singularity.
23	**Find the zeros of the function $f(z)=\tan z$ and its pole (NOV/DEC 2016)**BTL1 Given $f(z)=\tan z=\frac{\sin z}{\cos z}=\frac{P(z)}{Q(z)}$ The poles are given by $\cos z=0$ $z=(2n+1)\frac{\pi}{2}\quad where\quad n=0,\pm1,\pm2,\pm3,....$ $\text{Res}\left[f(z),a\right]=\frac{P(a)}{Q'(a)}$ Now $\frac{P(z)}{Q'(z)}=\frac{\sin z}{-\sin z}=-1$ $\text{Res}\left[f(z),(2n+1)\frac{\pi}{2}\right]=-1\quad where\quad n=0,\pm1,\pm2,\pm3,....$ Hence the residue of each pole is -1
24	**Find the zeros of the function $f(z)=\cot z$ and it's pole** BTL1 Given $f(z)=\cot z=\frac{\cos z}{\sin z}=\frac{P(z)}{Q(z)}$ The poles are given by $\sin z=0$ $z=n\pi\quad where\quad n=0,\pm1,\pm2,\pm3,....$ Residue of f(z) at $z=n\pi$ is $\frac{P[n\pi]}{Q'[n\pi]}$ $\frac{P(z)}{Q'(z)}=\frac{\cos z}{\cos z}$ $\frac{P(z)}{Q'(z)}=\frac{\cos(2n+1)\frac{\pi}{2}}{\cos(2n+1)\frac{\pi}{2}}=1\quad where\quad n=0,\pm1,\pm2,\pm3,....$
25	**Find residue of $f(z)=\frac{z^2}{(z-1)^2(z+2)}$ and at its simple pole.** BTL1 Given $f(z)=\frac{z^2}{(z-1)^2(z+2)}$ The poles of f(z) are given by $(z-1)^2(z+2)=0$

	$z=1$ is a pole of order 2 and $z=-2$ is a pole order 1[Simple pole] Residue of f(z) at z=-2: [simple Pole] $\text{Res}\left[f(z)\right]_{z=a}=\lim_{z\to a}(z-a)f(z)$ $\text{Res}\left[f(z)\right]_{z=-2}=\lim_{z\to -2}(z+2)\frac{z^2}{(z-1)^2(z+2)}=\lim_{z\to -2}\frac{z^2}{(z-1)^2}=\frac{4}{9}$
	PART-B
1.	**Use Cauchy's integral formula to evaluate** $\int_C \frac{\sin\pi z^2+\cos\pi z^2}{(z-1)(z-2)}dz$ **where C is the circle** $\|z\|=3$ **(MAY/JUNE 2016) (8 M)** BTL3 **Answer : Refer Page No.4.10- Dr.M.CHANDRASEKAR** • $\frac{\sin\pi z^2+\cos\pi z^2}{(z-1)(z-2)}=\frac{1}{(z-2)}-\frac{1}{(z-1)}$ **(2 M)** • $\int_C \frac{f(z)}{(z-a)}dz=2\pi i\, f(a)$ **(2 M)** • $\int_C \frac{\sin\pi z^2+\cos\pi z^2}{(z-1)(z-2)}dz=4\pi i$ **(4 M)**
2.	**Use Cauchy's integral formula to evaluate** $\int_C \frac{z+4}{(z^2+2z+5)}dz$ **where C is the circle** $\|z+1-i\|=3$ **(NOV/DEC 2006) (NOV/DEC 2014) (8 M)** BTL3 **Answer : Refer Page No.4.16- Dr.M.CHANDRASEKAR** • $\frac{z+4}{(z^2+2z+5)}=\frac{\left(\frac{3+2i}{4i}\right)}{z-(-1+2i)}+\frac{\left(\frac{3-2i}{-4i}\right)}{z-(-1-2i)}$ **(2 M)** • $\int_C \frac{f(z)}{(z-a)}dz=2\pi i\, f(a)$ **(2 M)** • $\int_C \frac{z+4}{(z^2+2z+5)}dz=\frac{\pi(3+2i)}{2}$ **(4 M)**
3.	**Use Cauchy's integral formula to evaluate** $\int_C \frac{z}{(z-1)(z-2)}dz$ **where C is the circle** $\|z-2\|=\frac{1}{2}$ **(MAY/JUNE 2015) (8 M)** BTL3 **Answer : Refer Page No.4.24- Dr.M.CHANDRASEKAR**

	• $\int_C \frac{f(z)}{(z-a)}dz = 2\pi i\ f(a)$ **(2 M)** • $\int_C \frac{z}{(z-1)(z-2)}dz = 4\pi i$ **(6 M)**
4.	**Use Cauchy's integral formula to evaluate** $\int_C \frac{z+1}{(z-3)(z-1)}dz$ **where C is the circle** $\lvert z\rvert = 2$ **(MAY/JUNE 2016) (8 M)** BTL3 **Answer : Refer Page No.4.29- Dr.M.CHANDRASEKAR** • $\int_C \frac{f(z)}{(z-a)}dz = 2\pi i\ f(a)$ **(2 M)** • $\int_C \frac{z+1}{(z-3)(z-1)}dz = -2\pi i$ **(6 M)**
5.	**Use Cauchy's integral formula to evaluate** $\int_C \frac{z-1}{(z-2)(z+1)^2}dz$ **where C is the circle** $\lvert z-i\rvert = 2$ **(8 M)** BTL3 **Answer : Refer Page No.4.31- Dr.M.CHANDRASEKAR** • $\int_C \frac{f(z)}{(z-a)^{n+1}}dz = \begin{cases} \frac{2\pi i}{n!} f^n(a) & ;\text{a lies inside c} \\ 0 & ;\text{a lies outside c} \end{cases}$ **(2 M)** • $\int_C \frac{z-1}{(z-2)(z+1)^2}dz = -\frac{2\pi i}{9}$ **(6 M)**
6.	**Use Cauchy's integral formula to evaluate** $\int_C \frac{z+1}{(z^2+2z+4)}dz$ **where C is the circle** $\lvert z+1+i\rvert = 2$ **(8 M)** BTL3 **Answer : Refer Page No.4.39- Dr.M.CHANDRASEKAR** • $\int_C \frac{f(z)}{(z-a)}dz = 2\pi i\ f(a)$ **(2 M)** • $\int_C \frac{z+1}{(z^2+2z+4)}dz = \pi i$ **(6 M)**

7.	**Expand** $\frac{z^2-1}{(z+2)(z+3)}$ **in the appropriate series in the regions** $(i)\, 2<\lvert z\rvert<3 \quad (ii)\, \lvert z\rvert>3$ **using Laurent's series. (8 M)** BTL2 **Answer : Refer Page No.4.51- Dr.M.CHANDRASEKAR** • $f(z)=1+\frac{3}{z+2}-\frac{8}{z+3}$ **(2 M)** (i) In $2<\lvert z\rvert<3,$ • $f(z)=1+\frac{3}{z}\sum_{n=0}^{\infty}(-1)^n\left(\frac{2}{z}\right)^n-\frac{8}{3}\sum_{n=0}^{\infty}(-1)^n\left(\frac{z}{3}\right)^n$ **(3 M)** (ii) In $\lvert z\rvert>3,$ • $f(z)=1+\frac{3}{z}\sum_{n=0}^{\infty}(-1)^n\left(\frac{2}{z}\right)^n-\frac{8}{z}\sum_{n=0}^{\infty}(-1)^n\left(\frac{3}{z}\right)^n$ **(3 M)**
8.	**Expand** $f(z)=\frac{7z-2}{z(z-2)(z+1)}$ **in Laurent's series in the regions** $(i)\, 2<\lvert z\rvert<3 \quad (ii)\, \lvert z\rvert>3$ **(8 M)** BTL2 **Answer : Refer Page No.4.52- Dr.M.CHANDRASEKAR** • $f(z)=\frac{1}{z}+\frac{2}{z-2}-\frac{3}{z+1}$ **(2 M)** (i) In $2<\lvert z\rvert<3,$ • $f(z)=\frac{1}{z}+\sum_{n=0}^{\infty}\left(\frac{2}{z}\right)^{n+1}+3\sum_{n=0}^{\infty}(-1)^{n+1}\left(\frac{1}{z}\right)^{n+1}$ **(3 M)** (ii) In $\lvert z\rvert>3,$ • $f(z)=\frac{1}{z}+\sum_{n=0}^{\infty}\left(\frac{2}{z}\right)^{n+1}+3\sum_{n=0}^{\infty}(-1)^{n+1}\left(\frac{1}{z}\right)^{n+1}$ **(3 M)**
9.	**Expand** $f(z)=\frac{7z-2}{z(z-2)(z+1)}$ **in Laurent's series in the region** $(i)\lvert z\rvert<2 \quad (ii)\, 1<\lvert z+1\rvert<3$ **(MAY/JUNE 2014) (8 M)** BTL2 **Answer : Refer Page No.4.52- Dr.M.CHANDRASEKAR** • $f(z)=\frac{1}{z}+\frac{2}{z-2}-\frac{3}{z+1}$ **(2 M)**

	(i) In $\lvert z\rvert < 2,$ • $f(z)=\frac{1}{z}-\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^{n}-3\sum_{n=0}^{\infty}\left(z\right)^{n}$ **(3 M)** (ii) In $1<\lvert z+1\rvert < 3,$ • $f(z)=\frac{-3}{z+1}+\sum_{n=1}^{\infty}\left(\frac{1}{z+1}\right)^{n}-\frac{2}{3}\sum_{n=0}^{\infty}\left(\frac{z+1}{3}\right)^{n}$ **(3 M)**
10.	**Expand** $f(z)=\frac{1}{(z-1)(z-2)}$ **in Laurent's series in the region** $(i)\lvert z\rvert > 2$ (ii) $0<\lvert z-1\rvert<1$ **(NOV/DEC 2014) (8 M)** BTL2 **Answer : Refer Page No.4.57- Dr.M.CHANDRASEKAR** • $f(z)=\frac{-1}{z-1}+\frac{1}{z-2}$ **(2 M)** (i) In $\lvert z\rvert > 2,$ • $f(z)=-\sum_{n=0}^{\infty}\left(\frac{1}{z}\right)^{n}+\frac{1}{z}\sum_{n=0}^{\infty}\left(\frac{2}{z}\right)^{n}$ **(3 M)** (ii) In $0<\lvert z-1\rvert<1,$ • $f(z)=\frac{-1}{z-1}+\sum_{n=0}^{\infty}\left(z-1\right)^{n}$ **(3 M)**
11.	**Use Cauchy's Residue theorem to evaluate** $\int_C \frac{\sin \pi z^2+\cos \pi z^2}{(z-1)^2(z-2)}dz$ **where C is the circle** $\lvert z\rvert = 3$ **(NOV/DEC 2015) (8 M)** BTL3 **Answer : Refer Page No.4.96- Dr.M.CHANDRASEKAR** • $\int_C f(z)dz = 2\pi i$ [sum of the residues] **(2 M)** • $\left\{\text{Res } f(z)_{at\, z=2}\right\}=1$ $\left\{\text{Res } f(z)_{at\, z=1}\right\}=-2\pi+1$ **(4 M)** • $\int_C \frac{\sin \pi z^2+\cos \pi z^2}{(z-1)^2(z-2)}dz = 4\pi i(1-\pi)$ **(2 M)**

12.	**Use Cauchy's Residue theorem to evaluate** $\int_C \frac{12z-7}{(z-1)^2(2z+3)}dz$ **where C is the circle** $\lvert z\rvert = 2$ **(8 M)** BTL3 **Answer : Refer Page No.4.92- Dr.M.CHANDRASEKAR** • $\int_C f(z)dz = 2\pi i$ [sum of the residues] **(2 M)** • $\left\{\text{Res } f(z)_{at\, z=-3/2}\right\} = -4$; $\left\{\text{Res } f(z)_{at\, z=1}\right\} = 4$ **(4 M)** • $\int_C \frac{12z-7}{(z-1)^2(2z+3)}dz = 0$ **(2 M)**
13.	**Use Cauchy's Residue theorem to evaluate** $\int_C \frac{z^2}{(z+1)^2(z^2+4)}dz$ **where C is the circle** $\lvert z\rvert = 3$ **(8 M)** BTL3 **Answer : Refer Page No.4.99- Dr.M.CHANDRASEKAR** • $\int_C f(z)dz = 2\pi i$ [sum of the residues] **(2 M)** • $\left\{\text{Res } f(z)_{at\, z=-1}\right\} = -\frac{8}{25}$; $\left\{\text{Res } f(z)_{at\, z=2i}\right\} = \frac{-4}{(1+2i)^2(4i)}$; $\left\{\text{Res } f(z)_{at\, z=-2i}\right\} = \frac{-4}{(1-2i)^2(-4i)}$ **(4 M)** • $\int_C \frac{z^2}{(z+1)^2(z^2+4)}dz = 0$ **(2 M)**
14.	**Use Cauchy's Residue theorem to evaluate** $\int_C \frac{dz}{(z^2+4)^2}$ **where C is the circle** $\lvert z-i\rvert = 2$ **(8 M)** BTL3 **Answer : Refer Page No.4.100- Dr.M.CHANDRASEKAR** • $\int_C f(z)dz = 2\pi i$ [sum of the residues] **(2 M)**

	• $\left\{\text{Res } f(z)_{at\, z=2i}\right\} = \dfrac{1}{32i}$ **(4 M)** $\left\{\text{Res } f(z)_{at\, z=-2i}\right\} = 0$ • $\int_C \dfrac{dz}{(z^2+4)^2} = \dfrac{\pi}{16}$ **(2 M)**
15.	**Evaluate** $\int_0^{2\pi} \dfrac{\cos 2\theta}{5+4\cos\theta} d\theta$ **by using Contour integration (MAY/JUNE 2014) (8 M)** BTL5 **Answer : Refer Page No.4.105- Dr.M.CHANDRASEKAR** • $\int_0^{2\pi} \dfrac{\cos 2\theta}{5+4\cos\theta} d\theta = \dfrac{1}{4i}\int_C \dfrac{(z^2+1)dz}{z^2(z+1/2)(z+2)}$ **(3 M)** • $\int_C f(z)dz = 2\pi i\ [\text{sum of the residues}]$ **(1 M)** $\left\{\text{Res } f(z)_{at\, z=0}\right\} = \dfrac{-5}{2}$ • $\left\{\text{Res } f(z)_{at\, z=-1/2}\right\} = \dfrac{17}{6}$ **(3 M)** $\left\{\text{Res } f(z)_{at\, z=-2}\right\} = 0$ • $\int_0^{2\pi} \dfrac{\cos 2\theta}{5+4\cos\theta} d\theta = \dfrac{\pi}{6}$ **(1 M)**
16.	**Prove that** $\int_0^{2\pi} \dfrac{d\theta}{5+4\sin\theta} = \dfrac{2\pi}{3}$ **by using Contour integration. (NOV/DEC 2006) (8 M)** BTL5 **Answer : Refer Page No.4.120- Dr.M.CHANDRASEKAR** • $\int_0^{2\pi} \dfrac{d\theta}{5+4\sin\theta} = \int_C \dfrac{dz}{(z+2i)(2z+i)}$ **(3 M)** • $\int_C f(z)dz = 2\pi i\ [\text{sum of the residues}]$ **(1 M)** $\left\{\text{Res } f(z)_{at\, z=-i/2}\right\} = \dfrac{1}{3i}$ • **(3 M)** $\left\{\text{Res } f(z)_{at\, z=-2i}\right\} = 0$

	• $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta} = \frac{2\pi}{3}$ **(1 M)**
17.	**Evaluate** $\int_0^{2\pi} \frac{d\theta}{13+5\sin\theta}$ **by using Contour integration. (NOV/DEC 2014) (8 M)** BTL5 **Answer : Refer Page No.4.123- Dr.M.CHANDRASEKAR** • $\int_0^{2\pi} \frac{d\theta}{13+5\sin\theta} = \int_C \frac{2dz}{(5z+i)(2+5i)}$ **(3 M)** • $\int_C f(z)dz = 2\pi i$ [sum of the residues] **(1 M)** • $\left\{\text{Res } f(z)_{at\, z=5i}\right\} = 0$ $\left\{\text{Res } f(z)_{at\, z=-\frac{i}{5}}\right\} = \frac{1}{12i}$ **(3 M)** • $\int_0^{2\pi} \frac{d\theta}{13+5\sin\theta} = \frac{\pi}{6}$ **(1 M)**
18.	**Evaluate** $\int_{-\infty}^{\infty} \frac{x^2dx}{(x^2+1)(x^2+4)}$ **by using Contour integration. (NOV/DEC 2008) (8 M)** BTL5 **Answer : Refer Page No.4.92- Dr.G.BALAJI** • $\int_{-\infty}^{\infty} \frac{x^2dx}{(x^2+1)(x^2+4)} = \int_C \frac{z^2}{(z^2+1)(z^2+4)}dz$ **(1 M)** • $\int_C f(z)dz = 2\pi i$ [sum of the residues] **(1 M)** • $\left\{\text{Res } f(z)_{at\, z=i}\right\} = \frac{i}{6}$ $\left\{\text{Res } f(z)_{at\, z=2i}\right\} = -\frac{i}{3}$ **(3 M)** • $\int_{-\infty}^{\infty} \frac{x^2dx}{(x^2+1)(x^2+4)} = \frac{\pi}{3}$ **(3 M)**
19.	**Evaluate** $\int_0^{\infty} \frac{\cos mx}{(x^2+a^2)}dx$ **by using Contour integration. (NOV/DEC 2016) (8 M)** BTL5 **Answer : Refer Page No.4.101- Dr.G.BALAJI**

	• $\int_0^\infty \frac{\cos mx\, dx}{(x^2+a^2)} = R.P\ of\ \int_C \frac{e^{mz}}{(z^2+a^2)}dz$ **(1 M)** • $\int_C f(z)dz = 2\pi i\ [\text{sum of the residues}]$ **(1 M)** • $\left\{\text{Res } f(z)_{at\, z=ai}\right\} = \frac{e^{-ma}}{2ai}$ **(3 M)** • $\int_0^\infty \frac{\cos mx}{(x^2+a^2)}dx = \frac{\pi e^{-ma}}{2a}$ **(3 M)**

	UNIT V LAPLACE TRANSFORMS
	Existence conditions – Transforms of elementary functions – Transform of unit step function and unit impulse function – Basic properties – Shifting theorems -Transforms of derivatives and integrals – Initial and final value theorems – Inverse transforms – Convolution theorem – Transform of periodic functions – Application to solution of linear second order ordinary differential equations with constant coefficients.
	PART * A
Q.No.	**Questions**
1.	**State the sufficient condition for the existence of Laplace transforms.** **(OR) State the conditions under which the Laplace Transform of $f(t)$ exisits.** **(APR/MAY 2015, 2017)** BTL1 The Laplace transform of $f(t)$ exists if a) $f(t)$ is piecewise continuous in $[a, b]$ where $a > 0$. b) $f(t)$ is of exponential order.
2.	**Is the linearity property applicable to $L\left[\frac{1-cost}{t}\right]$?Reason out?** BTL5 Given, $L\left[\frac{1-cost}{t}\right] = L\left[\frac{1}{t}\right] - L\left[\frac{cos\, t}{t}\right]$ by linearity property, provided the result exists. $L\left[\frac{1}{t}\right]$ does not exist. Since $\lim_{t\to 0}\frac{1}{t} = \frac{1}{0} = \infty$. $L\left[\frac{cos\, t}{t}\right]$ does not exist. Since, $\lim_{t\to 0}\frac{\cos t}{t} = \frac{1}{0} = \infty$. $\therefore$ Linearity property is not applicable to $L\left[\frac{1-cost}{t}\right]$.

3.	**If L[F(t)]=$F(s)$, Prove that $L\left[f\left(\frac{t}{5}\right)\right]=5F(5s)$.** BTL5 $L[f(t)]=\int_0^\infty e^{-st}f(t)dt$ $put\quad \frac{t}{5}=u \Rightarrow 5du=dt$ $L\left[f\left(\frac{t}{5}\right)\right]=\int_0^\infty e^{-(5s)u}f(u)5du$ $=5\int_0^\infty e^{-(5s)u}f(u)du=5F(5s)$
4	**Find the Laplace transform of unit step function.** BTL1 The unit step function is $u_a(t)=\begin{cases}0 & t<a\\ 1 & t>a,\end{cases}\quad a\geq 0$ The Laplace transform $L[f(t)]=\int_0^\infty e^{-st}f(t)dt=\int_a^\infty e^{-st}(1)dt=\left[\frac{e^{-st}}{-s}\right]_a^\infty=-\frac{1}{s}\left[e^{-\infty}-e^{-as}\right]=\frac{e^{-as}}{s}$.
5	**Prove that** $L\left(\int_0^t f(t)dt\right)=\frac{F(s)}{s}$ **where $L[f(t)]=F(s)$. [DEC 2016]** BTL5 Let $F(t)=\int_0^t f(t)dt$ $F'(t)=f(t)$ $L[F'(t)]=sL[F(t)]-F(0)=sL[F(t)]-0$ $L[f(t)]=sL[F(t)]=sL[\int_0^t f(t)dt]$ $\therefore L\left(\int_0^t f(t)dt\right)=\frac{F(s)}{s}$
6	**Does** $L\left[\frac{\cos at}{t}\right]$ **exist?** BTL4 $\underset{t\to 0}{Lt}\frac{f(t)}{t}=\underset{t\to 0}{Lt}\frac{\cos at}{t}=\frac{1}{0}=\infty$ $\therefore L\left[\frac{\cos at}{t}\right] does \quad not \quad exist.$
7	**Obtain the Laplace transform of $sin2t-2tcos2t$.** BTL3

	$L[\sin 2t - 2t\cos 2t] = L[\sin 2t] - 2L[t\cos 2t] = L[\sin 2t] - 2\left(-\frac{d}{ds}L[\cos 2t]\right)$ $= \frac{2}{s^2+4} + 2\frac{d}{ds}\left(\frac{s}{s^2+4}\right) = \frac{2}{s^2+4} + 2\left(\frac{(s^2+4)(1)-s(2s)}{(s^2+4)^2}\right)$ $= \frac{2(s^2+4)+2(4-s^2)}{(s^2+4)^2} = \frac{16}{(s^2+4)^2}$.
8	**Find** $L^{-1}\left[\frac{s+2}{s^2+2s+2}\right]$**.** BTL4 $L^{-1}\left[\frac{s+2}{s^2+2s+2}\right] = L^{-1}\left[\frac{(s+1)+1}{(s+1)^2+1}\right] \{\because L^{-1}[F(s+a)] = e^{-at}L^{-1}[F(s)]\}$ $= L^{-1}\left[\frac{(s+1)}{(s+1)^2+1}\right] + L^{-1}\left[\frac{1}{(s+1)^2+1}\right]$ $= e^{-t}\left(L^{-1}\left[\frac{s}{s^2+1}\right] + L^{-1}\left[\frac{1}{s^2+1}\right]\right)$ $= e^{-t}(\cos t + \sin t)$.
9	**What is the Laplace transform of $f(t), 0 < t < 10$ with $f(t) = f(t+10)$**? BTL3 Given $f(t)$ is a periodic function with period p. $L[f(t)] = \frac{1}{1-e^{-ps}}\int_0^p e^{-st}f(t)dt$ $put \quad p = 10, \quad L[f(t)] = \frac{1}{1-e^{-10s}}\int_0^{10} e^{-st}f(t)dt$
10	**State and Prove Linearity property. [MAY/JUNE 2016]** BTL1 $Statement: \quad L[af(t) \pm bg(t)] = aL[f(t)] \pm bL[g(t)]$ $proof: \quad L[f(t)] = \int_0^\infty e^{-st}f(t)dt$ $L[af(t) \pm bg(t)] = \int_0^\infty e^{-st}L[af(t) \pm bg(t)]dt$ $= \int_0^\infty e^{-st}af(t)dt \pm \int_0^\infty e^{-st}bg(t)dt$ $= a\int_0^\infty e^{-st}f(t)dt \pm b\int_0^\infty e^{-st}g(t)dt$ $= aL[f(t)] \pm bL[g(t)]$.
11	**Find** $L^{-1}\left(\frac{S}{S^2+4S+5}\right)$**. [MAY/JUNE 2016]** BTL3

	$L^{-1}\left(\frac{S}{S^2+4S+5}\right)=L^{-1}\left(\frac{(S+2)-2}{(S+2)^2+1}\right)=e^{-2t}L^{-1}\left(\frac{S-2}{S^2+1}\right)$ $=e^{-2t}\left[L^{-1}\left(\frac{S-2}{S^2+1}\right)-2L^{-1}\left(\frac{1}{S^2+1}\right)\right]=e^{-2t}[\cos t-2\sin t].$
12	**Find** $L[te^{-3t}\cos 2t]$**.** BTL3 We know that $L[t\cos at]=\frac{s^2-a^2}{(s^2+a^2)^2}$, $L[te^{-3t}\cos 2t]=\left[\frac{s^2-2^2}{(s^2+2^2)^2}\right]_{s\to s+3}=\frac{(s+3)^2-2^2}{((s+3)^2+2^2)^2}$
13	**Find** $L^{-1}\left[\tan^{-1}\left(\frac{1}{s}\right)\right]$**.** BTL3 *Let* $F(s)=\tan^{-1}\left(\frac{1}{s}\right)$ $F'(s)=\frac{1}{1+\left(1/s\right)^2}\left(\frac{-1}{s^2}\right)=\frac{-1}{s^2+1}$ *By property* $L^{-1}[F'(s)]=-L^{-1}\left[\frac{1}{s^2+1}\right]=-\sin t$ $\therefore L^{-1}[F'(s)]=-\sin t;$ $L^{-1}[F(s)]=\frac{-1}{t}L^{-1}[F'(s)]$ $L^{-1}\left[\tan^{-1}\left(\frac{1}{s}\right)\right]=\frac{\sin t}{t}.$
14	**Solve using Laplace transform** $\frac{dy}{dt}+y=e^{-t}$ **given that** $y(0)=0$. BTL3 Taking Laplace transform on both sides, we get $L[y'(t)]+L[y(t)]=L[e^{-t}]$ $sL[y(t)]-y(0)+L[y(t)]=L[e^{-t}]$ $sL[y(t)]-0+L[y(t)]=\frac{1}{s+1}$

	$(s+1)L[y(t)] = \frac{1}{s+1}$ $L[y(t)] = \left(\frac{1}{(s+1)^2}\right)$ $\therefore\ y(t) = L^{-1}\left(\frac{1}{(s+1)^2}\right) = e^{-t}L\left(\frac{1}{s^{2]gh}[\ ]}\right) = e^{-t}t.$ $\{\because L[e^{-at}f(t)] = F(s+a)\}$
15	**Given an example for a function that do not have Laplace transform.** BTL5 Consider $\boldsymbol{f(t) = e^{t^2}}$, since $\underset{t\to\infty}{Lt}\, e^{-st}e^{t^2} = \infty$, hence $\boldsymbol{e^{t^2}}$ is not exponential order. Hence $\boldsymbol{f(t) = e^{t^2}}$ does not have Laplace transform.
16	**Can $\boldsymbol{F(s)} = \frac{s^3}{(s+1)^2}$ be the Laplace transform of some $\boldsymbol{f(t)}$?** BTL5 $\underset{s\to\infty}{Lt}\, F(s) = \underset{s\to\infty}{Lt}\, \frac{s^3}{(s+1)^2} \neq 0$ Hence $\boldsymbol{F(s)}$ cannot be Laplace transform of $\boldsymbol{f(t)}$.
17	**Evaluate $\int_0^t \sin u\cos(t-u)du$ using Laplace Transform.** BTL3 $Let\quad L\left[\int_0^t \sin u\cos(t-u)du\right] = L[\sin t * \cos t]$ $= L[\sin t]L[\cos t] \qquad (by\quad convolution\quad theorem)$ $= \frac{1}{(s^2+1)}\frac{s}{(s^2+1)} = \frac{s}{(s^2+1)^2}.$ $\int_0^t \sin u\cos(t-u)du = L^{-1}\left[\frac{s}{(s^2+1)^2}\right] = \frac{1}{2}L^{-1}\left[\frac{2s}{(s^2+1)^2}\right] = \frac{t}{2}\sin t.$ $\left[\because L^{-1}\left(\frac{2s}{(s^2+1)^2}\right) = t\sin at\right].$
18	**Given an example for a function having Laplace transform but not satisfying the continuity condition. BTL1** $f(t) = t^{\frac{-1}{2}}$ has Laplace transform even though it does not satisfy the continuity condition. (i.e.) It is not piecewise continuous in $(0,\infty)$ $\quad as\ \underset{t\to 0}{Lt}\, f(t) = \infty.$
19	**Define a Periodic function with example.** BTL1 $\boldsymbol{f(t)}$ for all $\boldsymbol{t}$. The least value of $\boldsymbol{p > 0}$ is called the period of $\boldsymbol{f(t)}$. For example, $\boldsymbol{\sin t}$ and $\boldsymbol{\cos t}$ are periodic functions with period $2\boldsymbol{\pi}$.
20	**If $\boldsymbol{L[f(t)] = F(s)}$, find $\boldsymbol{L[f(at)]}$. [APR/MAY 2018]** BTL5

	$L[f(at)]=\int_0^\infty e^{-st} f(at)dt$ $put \quad u=at$ $L[f(at)]=\int_0^\infty e^{-\left(\frac{s}{a}\right)u} f(u)\frac{du}{a}=\frac{1}{a}\int_0^\infty e^{-\left(\frac{s}{a}\right)u} f(u)du=\frac{1}{a}F\left(\frac{s}{a}\right).$
21	**Find the Laplace transform of $\frac{t}{e^t}$. [APR/MAY 2018]** BTL3 $L\left[\frac{t}{e^t}\right]=L[e^{-t}t]=\left[\frac{1}{s^2}\right]_{s\to s+1}=\frac{1}{(s+1)^2}.$
22	**State Convolution theorem on Laplace Transform. [MAY/JUNE 2017]** BTL1 The Laplace transform of convolution of two functions is equal to the product of their Laplace transform. (i.e) $L[f(t)*g(t)]=L[f(t)]L[g(t)]$.
23	**Find $L\left[\frac{1}{\sqrt{t}}\right]$. [APR/MAY 2017]** BTL3 We know that, $L[t^n]=\frac{\Gamma(n+1)}{s^{n+1}}$ $L\left[\frac{1}{\sqrt{t}}\right]=L[t^{-1/2}]$ $=\frac{\Gamma(-1/2+1)}{s^{-1/2+1}}$ $=\frac{\Gamma(1/2)}{s^{1/2}}=\sqrt{\frac{\pi}{s}}.$
24	**Find the Laplace transform $sin^3(2t)$.** BTL3 $L[\sin^3(2t)]=\frac{1}{4}L[3\sin 2t-\sin 6t]$ $=\frac{3}{4}L[\sin 2t]-\frac{1}{4}L[\sin 6t]$ $\{\because \sin^3 t=\frac{1}{4}[3\sin t-\sin 3t]\}$ $=\frac{3}{4}\left(\frac{2}{s^2+4}\right)-\frac{1}{4}\left(\frac{6}{s^2+36}\right)$ $=\frac{6}{4}\left\{\left(\frac{1}{s^2+4}\right)-\left(\frac{1}{s^2+36}\right)\right\}$
25	**Find the Laplace transform of $e^{-2t}t^{1/2}$.** BTL3

	$L(e^{-2t}t^{1/2}) = L[t^{1/2}]_{s\to s+2}$ $\because if \quad L[f(t)] = F(s), \quad then \quad l[e^{-at}f(t)] = F(s)/_{s\to s+2}$ $\left[\dfrac{\Gamma\left(\frac{1}{2}+1\right)}{s^{\frac{3}{2}}}\right]_{s\to s+2} = \left[\dfrac{\frac{1}{2}\Gamma\left(\frac{1}{2}\right)}{s^{\frac{3}{2}}}\right]_{s\to s+2}$ $= \dfrac{\frac{1}{2}\sqrt{\pi}}{(s+2)^{\frac{3}{2}}} \qquad \left(\because \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}, \quad \Gamma n+1 = n\Gamma n\right).$
26	**Does** $L\left[\dfrac{\cos at}{t}\right]$ **exist?** BTL5 $\underset{t\to 0}{Lt}\dfrac{f(t)}{t} = \underset{t\to 0}{Lt}\dfrac{\cos at}{t} = \dfrac{1}{0} = \infty$ $\therefore L\left[\dfrac{\cos at}{t}\right] does \quad not \quad exist.$
27	**Using Laplace transform, Evaluate** $\int_0^\infty te^{-2t}\sin t dt$. **[APR/MAY 2015]** BTL3 $\int_0^\infty e^{-2t}f(t)dt = \left[\int_0^\infty e^{-st}f(t)dt\right]_{s=2} = [L[t\sin t]]_{s=2} = \left[-\dfrac{d}{ds}L[\sin t]\right]_{s=2} = -\dfrac{d}{ds}\left(\dfrac{1}{s^2+1}\right) = \dfrac{4}{25}$
	Part*B
1	**Find** **1)** $L[\frac{\sinh 2t}{t}]$. **2)** $L[\frac{e^{-t}\sin t}{t}]$ **3)** $L[\frac{\cos at-\cos bt}{t}]$. **[APR/MAY 2011,2015, NOV/DEC 2012,2016] (12M)** BTL3 **Answer: Refer Page No:5.35 - Dr. G. Balaji.** 1) $L\left[\dfrac{\sinh 2t}{t}\right] = \int_s^\infty L[\sinh 2t]ds = \int_s^\infty \dfrac{2}{s^2-4}ds = 2\left[\dfrac{1}{2(2)}\log\left(\dfrac{s-2}{s+2}\right)\right]_s^\infty$ $= \dfrac{1}{2}\left[\log\dfrac{s+2}{s-2}\right] = \log\sqrt{s+2/_{s-2}} \qquad (4M)$ 2)

	$L\left[\frac{e^{-t}\sin t}{t}\right] = \left[L\left[\sin t/t\right]\right]_{s\to s+1}$ $= \left[\cot^{-1} s\right]_{s\to(s+1)} = \cot^{-1}(s+1).$ (3M) 3) $L\left[\frac{\cos at - \cos bt}{t}\right] = \int_s^\infty L[\cos at - \cos bt]ds$ $= \int_s^\infty \left[\frac{s}{s^2+a^2} - \frac{s}{s^2+b^2}\right]ds = \frac{1}{2}\left[\log(s^2+a^2) - \log(s^2+b^2)\right]_s^\infty = \frac{1}{2}\log\frac{s^2+b^2}{s^2+a^2}.$ (5M)
2	**1) State and prove Initial Value and Final value theorem. [APR/MAY 2017]** **2) Verify the initial and Final value theorem for $f(t) = 1 + e^t(\sin t + \cos t)$. [NOV/DEC 2009, MAY/JUNE 2012]** **3) Using the initial value theorem, find $\underset{s\to\infty}{Lt}\, sL[f(t)]$ for the function $f(t) = e^{-t}\cos t$.** **[NOV/DEC 2016] (16M)** BTL3 **Answer: Refer Page No:5.40 - Dr. G. Balaji.** 1) Initial Value theorem Statement: $L[f(t)] = F(s),\ then\ \underset{t\to 0}{Lt}\, f(t) = \underset{s\to\infty}{Lt}\, sF(s).$ $Proof: We\ know\ that\ L[f'(t)] = sL[f(t)] - f(0) = sF(s) - f(0)$ $= \int_0^\infty e^{-st} f'(t)dt$ $\underset{s\to\infty}{Lt}\left[sF(s) - f(0)\right] = \underset{s\to\infty}{Lt}\int_0^\infty e^{-st} f'(t)dt = \underset{s\to\infty}{Lt}\, sF(s) - f(0) = 0$ $hence\ \underset{t\to 0}{Lt}\, f(t) = \underset{s\to\infty}{Lt}\, sF(s).$ (2M) Final Value theorem Statement: $L[f(t)] = F(s),\ then\ \underset{t\to\infty}{Lt}\, f(t) = \underset{s\to 0}{Lt}\, sF(s).$ $Proof: We\ know\ that\ l[f'(t)] = sL[f(t)] - f(0) = sF(s) - f(0)$ $= \int_0^\infty e^{-st} f'(t)dt$ $\underset{s\to 0}{Lt}\left[sF(s) - f(0)\right] = \underset{s\to 0}{Lt}\int_0^\infty e^{-st} f'(t)dt = \underset{s\to 0}{Lt}\, sF(s) - f(0) = f(\infty) - f(0)$ $hence\ \underset{t\to\infty}{Lt}\, f(t) = \underset{s\to 0}{Lt}\, sF(s).$ (2M) 2) $f(t) = 1 + e^t(\sin t + \cos t)$ Initial Value theorem state that $L[f(t)] = F(s),\ then\ \underset{t\to 0}{Lt}\, f(t) = \underset{s\to\infty}{Lt}\, sF(s).$

	$L[f(t)] = L[1+e^{t}(\sin t+\cos t)]$ $=\frac{1}{s}+\frac{1}{(s+1)^2+1}+\frac{s+1}{(s+1)^2+1}$ $LHS \;=\lim_{t\to 0} f(t)=2.$ $RHS = \underset{s\to\infty}{Lim}\left[1+\frac{s(s+2)}{(s+1)^2+1}\right]=2 \qquad (4M)$ $LHS \;=\; RHS$ Hence, Initial Value theorem verified. Final Value theorem state that $L[f(t)]=F(s), \quad then \quad \underset{t\to\infty}{Lt}\, f(t) = \underset{s\to 0}{Lt}\, sF(s).$ $LHS \;=\lim_{t\to\infty} f(t)=1.$ $RHS = \underset{s\to 0}{Lim}\left[1+\frac{s(s+2)}{(s+1)^2+1}\right]=1 \qquad (4M)$ $LHS \;=\; RHS$ 3) <u>Initial Value theorem Statement:</u> $L[f(t)]=F(s), \quad then \quad \underset{t\to 0}{Lt}\, f(t) = \underset{s\to\infty}{Lt}\, sF(s).$ $f(t)=e^{-t}\cos t$ $\lim_{t\to 0} f(t)=1$ $\lim_{s\to\infty} sF(s)=1 \qquad (4M)$ Hence proved.
3	**Using convolution theorem find** $L^{-1}\left[\frac{1}{(s+a)(s+b)}\right]$ **. [APR/MAY 2011] (8M)** BTL3 **Answer: Refer Page No:5.77- Dr. G. Balaji.** $L^{-1}\left[\frac{1}{(s+a)(s+b)}\right]=L^{-1}\left[\left(\frac{1}{(s+a)}\right)\left(\frac{1}{(s+b)}\right)\right]$ $=L^{-1}\left(\frac{1}{(s+a)}\right)*L^{-1}\left(\frac{1}{(s+b)}\right)$ $=e^{-at}*e^{-bt} \qquad (3M)$ $=\int_0^t e^{-at}e^{-b(t-u)}du$

	$= e^{-bt}\left[\frac{e^{-(a-b)u}}{-(a-b)}\right]_{u=0}^{u=t}$ $(3M)$ $= \frac{e^{-bt}-e^{-at}}{a-b}.$ $(2M)$ Note: **Using convolution theorem find** $L^{-1}\left[\frac{1}{(s+1)(s+2)}\right]$**. [NOV/DEC 2007,2012] (8M)** **Hint:** In the above problem put $a = 2, b = 1$.
4	**Find the Laplace inverse of** $\left[\frac{s^2}{(s^2+a^2)^2}\right]$ **using convolution theorem. [NOV/DEC 2011] (8M)** BTL3 **Answer: Refer Page No:5.84- Dr. G. Balaji.** $L^{-1}\left[\frac{s^2}{(s^2+a^2)^2}\right] = L^{-1}\left[\left(\frac{s}{(s^2+a^2)}\right)\left(\frac{s}{(s^2+a^2)}\right)\right]$ $= L^{-1}\left(\frac{s}{(s^2+a^2)}\right) * L^{-1}\left(\frac{s}{(s^2+a^2)}\right)$ $= \cos at * \cos at$ $(3M)$ $= \int_0^t \cos au \cos a(t-u)\,du$ $= \frac{1}{2}\int_0^t [\cos(au+at-au)+\cos(au-at+au)]du$ $(2M)$ $= \frac{1}{2}\left[[(\cos at)u] + \left[\frac{\sin[2au-at]}{2a}\right]\right]_{u=0}^{u=t}$ $= \frac{1}{2}\left[t\cos at + \frac{\sin at}{a}\right]$ $L^{-1}\left[\frac{s^2}{(s^2+a^2)^2}\right] = \frac{1}{2a}[\sin at + at\cos at].$ $(3M)$ Note: **Using Convolution theorem, find** $L^{-1}\left[\frac{s^2}{(s^2+4)^2}\right]$**. [NOV/DEC 2012] (8M)**

	Hint: In the problem put $a = 2$.
5	**Using convolution theorem find** $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$. **[NOV/DEC 2013, APR/MAY 2017] (8M)** BTL3 **Answer: Refer Page No:5.83- Dr. G. Balaji.** $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right] = L^{-1}\left[\left(\frac{s}{(s^2+a^2)}\right)\left(\frac{1}{(s^2+a^2)}\right)\right]$ $= L^{-1}\left(\frac{s}{(s^2+a^2)}\right) * \frac{1}{a} L^{-1}\left(\frac{a}{(s^2+a^2)}\right)$ $= \cos at * \frac{1}{a}\sin at \qquad (3M)$ $= \frac{1}{a}\int_0^t \cos au \sin a(t-u)du$ $= \frac{1}{2a}\int_0^t [\sin(at-au+au) + \sin(at-au-au)]du \qquad (2M)$ $= \frac{1}{2a}\left[\left[(\sin at)u\right] + \left[\frac{-\cos[a(t-2u)]}{-2a}\right]\right]_0^t$ $= \frac{1}{2a}\left[t\sin at + \frac{\cos at}{2a} - \frac{\cos at}{2a}\right]$ $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right] = \frac{1}{2a}t\sin at. \qquad (3M)$
6	**Using convolution theorem find** $L^{-1}\left[\frac{s}{(s^2+a^2)(s^2+b^2)}\right]$**. [MAY/JUNE 2016] (8M)** BTL3 **Answer: Refer Page No:5.81- Dr. G. Balaji.**

	$L^{-1}\left[\frac{s}{(s^2+a^2)(s^2+b^2)}\right]=L^{-1}\left[\left(\frac{s}{(s^2+a^2)}\right)\left(\frac{1}{(s^2+b^2)}\right)\right]$ $=L^{-1}\left(\frac{s}{(s^2+a^2)}\right)*L^{-1}\left(\frac{1}{(s^2+b^2)}\right)$ $=\cos at*\frac{1}{b}\sin bt \qquad (3M)$ $=\frac{1}{b}\int_0^t \cos au \sin b(t-u)du$ $=\frac{1}{2b}\int_0^t [\sin(au+bt-bu)+\sin(bt-bu-au)]du \qquad (2M)$ $=\frac{1}{2b}\left[\left[\frac{-\cos[(a-b)u+bt]}{a-b}\right]+\left[\frac{-\cos[bt-(a+b)u]}{-(a+b)}\right]\right]_0^t$ $=\frac{1}{2b}\left[\cos at\left(\frac{1}{a+b}-\frac{1}{a-b}\right)-\cos bt\left(\frac{1}{a+b}-\frac{1}{a-b}\right)\right]$ $L^{-1}\left[\frac{s}{(s^2+a^2)(s^2+b^2)}\right]=\frac{\cos at-\cos bt}{b^2-a^2}. \qquad (3M)$ <u>Note:</u> **Using convolution theorem find** $L^{-1}\left[\frac{s}{(s^2+1)(s^2+4)}\right]$**.** [**MAY/JUNE 2015,2016] (8M)** **Hint:** In the above problem put $a=1, b=2$, **Using convolution theorem find** $L^{-1}\left[\frac{s}{(s^2+4)(s^2+9)}\right]$**.** [**MAY/JUNE 2015,2016] (8M)** **Hint:** In the above problem put $=2, b=3$.
7	**Find** $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$ **using convolution theorem. [APR/MAY 2014, 2015,2016, NOV/DEC 2014, 2016] (8M)** BTL3

	Answer: Refer Page No:5.86- Dr. G. Balaji. $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right] = L^{-1}\left[\left(\frac{s}{(s^2+a^2)}\right)\left(\frac{s}{(s^2+b^2)}\right)\right]$ $= L^{-1}\left(\frac{s}{(s^2+a^2)}\right) * L^{-1}\left(\frac{s}{(s^2+b^2)}\right)$ $= \cos at * \cos bt \qquad (3M)$ $= \int_0^t \cos au \cos b(t-u) du$ $= \frac{1}{2}\int_0^t [\cos(au+bt-bu)+\cos(au-bt+bu)]du \qquad (2M)$ $= \frac{1}{2}\left[\left[\frac{\sin[(a-b)u+bt}{a-b}\right]+\left[\frac{\sin[(a+b)u-bt}{a+b}\right]\right]_0^t$ $= \frac{1}{2}\left[\sin at\left(\frac{1}{a-b}+\frac{1}{a+b}\right)+\sin bt\left(\frac{1}{a+b}-\frac{1}{a-b}\right)\right]$ $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right] = \frac{a\sin at - b\sin bt}{a^2-b^2}. \qquad (3M)$ Note: **Find** $L^{-1}\left[\frac{s^2}{(s^2+1)(s^2+4)}\right]$ **using convolution theorem. [APR/MAY 2017] (8M)** **Hint:** In the above problem put $a = 1$ & $b = 2$.
8	**Find the Laplace transform of the rectangular wave given by** $f(t) = \begin{cases} k & ,0<t<b \\ -k & ,b<t<2b \end{cases}$. **[APR/MAY 2008, 2015] (8M)** BTL5 **Answer: Refer Page No:5.92- Dr. G. Balaji.** Given, $f(t) = \begin{cases} k & ,0<t<b \\ -k & ,b<t<2b \end{cases}$. This function is periodic in the interval $(0,2b)$ with period $2b$.

	$L[f(t)] = \frac{1}{1-e^{-ps}}\int_0^p e^{-st} f(t)dt$ $L[f(t)] = \frac{1}{1-e^{-2bs}}\int_0^{2b} e^{-st} f(t)dt$ $= \frac{1}{1-e^{-2bs}}\left[\int_0^b e^{-st}(k)dt + \int_b^{2b} e^{-st}(-k)dt\right]$ $(2M)$ $= \frac{k}{1-e^{-2bs}}\left[\left[\frac{e^{-st}}{-s}\right]_0^b - \left[\frac{e^{-st}}{-s}\right]_b^{2b}\right]$ $(2M)$ $= \frac{k}{s}\frac{1}{1-e^{-2bs}}\left[1-2e^{-bs}+e^{-2bs}\right]$ $= \frac{k}{s}\frac{\left[1-e^{-bs}\right]^2}{\left(1-e^{-bs}\right)\left(1+e^{-bs}\right)}$ $(2M)$ $= \frac{k}{s}\tanh\left[\frac{bs}{2}\right]$ $(2M)$ Note: **Find the Laplace transform of the rectangular wave given by** $f(t) = \begin{cases} 1 & ,0<t<b \\ -1 & ,b<t<2b \end{cases}$. **[APR/MAY 2013, 2014] (8M)** **Hint:** In the above problem put $k = 1$. **Find the Laplace transform of the rectangular wave given by** $f(t) = \begin{cases} E & ,0<t<a \\ -E & ,a<t<2a \end{cases}$ **for all** $\boldsymbol{f(t+2a) = f(t)}$ **[NOV/DEC 2010] (8M)** **Hint:** In that above solved problem put $k = E$ and $b = a$.
9	**Find the Laplace transform of a square wave function given by** $f(t) = \begin{cases} E & for \quad 0 \le t \le a/2 \\ -E & for \quad a/2 \le t \le a \end{cases}$ **and** $\boldsymbol{f(t+a) = f(t)}$. **[NOV/DEC 2011, 2016, MAY/JUNE 2016] (8M)** BTL5 **Answer: Refer Page No:5.95- Dr. G. Balaji.**

	$L[f(t)]=\frac{1}{1-e^{-ps}}\int_0^p e^{-st}f(t)dt$ $L[f(t)]=\frac{1}{1-e^{-as}}\int_0^a e^{-st}f(t)dt$ $=\frac{1}{1-e^{-as}}\left[\int_0^{a/2} e^{-st}(E)dt+\int_{a/2}^{a} e^{-st}(-E)dt\right] \quad (2M)$ $=\frac{E}{1-e^{-as}}\left[\left[\frac{e^{-st}}{-s}\right]_0^{a/2}-\left[\frac{e^{-st}}{-s}\right]_{a/2}^{a}\right] \quad (2M)$ $=\frac{E}{s}\frac{1}{1-e^{-as}}\left[1-2e^{-as/2}+e^{-sa}\right]$ $=\frac{E}{s}\frac{\left[1-e^{-as/2}\right]^2}{\left(1-e^{-as/2}\right)\left(1+e^{-as/2}\right)} \quad (2M)$ $=\frac{E}{s}\tanh\left[\frac{as}{4}\right] \quad (2M)$
10	**Find the Laplace Transform of triangular wave function** $\begin{cases} t & , \quad 0<t<a \\ 2a-t & , \quad a<t<2a \end{cases}$ **with** $f(t+2a)=f(t)$. **[APR/MAY 2000, 2008, 2015, 2016, MAY/JUNE 2006, 2009, 2012, NOV/DEC 2005, 2009, 2014] (8M)** BTL5 **Answer: Refer Page No:5.94- Dr. G. Balaji.** $L[f(t)]=\frac{1}{1-e^{-2as}}\int_0^{2a} e^{-st}f(t)dt$ $=\frac{1}{1-e^{-2as}}\left[\int_0^a e^{-st}t\,dt+\int_a^{2a} e^{-st}(2a-t)dt\right] \quad (2M)$ $L[f(t)]=\frac{1}{1-e^{-2as}}\left[\frac{-ae^{-as}}{s}-\frac{e^{-as}}{s^2}+\frac{1}{s^2}+\frac{ae^{-as}}{s}+\frac{e^{-2as}}{s^2}-\frac{e^{-as}}{s^2}\right] \quad (3M)$

	$L[f(t)] = \frac{1}{1-e^{-2as}}\left[\frac{1-2e^{-as}+e^{-2as}}{s^2}\right]$ $= \frac{1}{s^2}\frac{(1-e^{-as})^2}{(1-e^{-as})(1+e^{-as})}$ $= \frac{1}{s^2}\frac{(1-e^{-as})}{(1+e^{-as})}.$ $= \frac{1}{s^2}\tanh\left[\frac{as}{2}\right].$ $(3M)$
11	**Using Laplace transform technique, solve $y'' + y' = t^2 + 2t$, given $y = 4, y' = -2$ when $t = 0$. [NOV/DEC 2013, MAY/JUNE 2016] (8M)** BTL 3 **Answer: Refer Page No:5.109- Dr. G. Balaji.** Given: $y'' + y' = t^2 + 2t$, $y = 4, y' = -2$ when $t = 0$, $L[y''(t)] + L[y'(t)] = L[t^2] + 2L[t]$ $s^2L[y(t)] - sy(0) - y'(0) + sL[y(t)] - y(0) = \frac{2}{s^3} + 2\frac{1}{s^2}$ $(2M)$ $(s^2+s)L[y(t)] = 4s + 2 + \frac{2+2s}{s^3} = \frac{4s^4+2s^3+2+2s}{s^3}$ $L[y(t)] = \frac{4s^4+2s^3+2+2s}{s^3(s^2+s)}$ $L[y(t)] = \frac{4}{s+1} + \frac{2}{s(s+1)} + \frac{2}{s^4}$ $(3M)$ $L[y(t)] = \frac{2}{s} + \frac{2}{s+1} + \frac{2}{s^4}$ $y(t) = 2L^{-1}\left[\frac{1}{s}\right] + 2L^{-1}\left[\frac{1}{s+1}\right] + 2L^{-1}\left[\frac{1}{s^4}\right]$ $y(t) = 2 + 2e^{-t} + \frac{1}{3}t^3.$ $(3M)$
12	**Solve $\frac{d^2y}{dt^2} + 4y = \sin 2t$, given $y(0) = 3$, and $y'(0) = 4$. [MAY/JUNE 2014] (8M)** BTL 3 **Answer: Refer Page No:5.106- Dr. G. Balaji.** Given: $\frac{d^2y}{dt^2} + 4y = \sin 2t$, $y(0) = 3$, and $y'(0) = 4$.

	$L[y''(t)]+4L[y(t)]=L[\sin 2t]$ $[s^2L[y(t)]-sy(0)-y'(0)]+4L[y(t)]=\frac{2}{s^2+4}$ $[s^2+4]L[y(t)]=\frac{2}{s^2+4}+3s+4 \qquad (3M)$ $L[y(t)]=\frac{2}{(s^2+4)^2}+\frac{3s}{s^2+4}+\frac{4}{s^2+4}$ $y(t)=\frac{2}{8}L^{-1}\left[\frac{(s^2+2^2)-(s^2-2^2)}{(s^2+2^2)^2}\right]+3\cos 2t+\frac{4}{2}\sin 2t. \qquad (3M)$ $y(t)=\frac{1}{8}\sin 2t-\frac{1}{4}t\cos 2t+3\cos 2t+2\sin 2t. \qquad (2M)$
13	**Solve $\frac{d^2x}{dt^2}-3\frac{dx}{dt}+2x=2$ given $x=0$ and $\frac{dx}{dt}=5$ for $t=0$ using Laplace transform method. [APR/MAY 2011, NOV/ DEC 2012] (8M)** BTL 3 **Answer: Refer Page No:5.100- Dr. G. Balaji.** Given: $\frac{d^2x}{dt^2}-3\frac{dx}{dt}+2x=2$ given $x=0$ and $\frac{dx}{dt}=5$ for $t=0$. $L[x''(t)]-3L[x'(t)]+2L[x(t)]=L[2]$ $[s^2L[x(t)]-sx(0)-x'(0)]-3[sL[x(t)]-x(0)]+2L[x(t)]=2L[1]$ $[s^2-3s+2]L[x(t)]=\frac{2}{s}+5$ $L[x(t)]=\frac{2+5s}{s(s^2-3s+2)} \qquad (2M)$ $L[x(t)]=\frac{1}{s}+\frac{(-7)}{s-1}+\frac{6}{(s-2)}$ $x(t)=L^{-1}\left[\frac{1}{s}\right]-7L^{-1}\left[\frac{1}{s-1}\right]+6L^{-1}\left[\frac{1}{(s-2)}\right] \qquad (3M)$ $x(t)=1-7e^{t}+6e^{2t} \qquad (3M)$
14	**Solve using Laplace transform, $x''-2x'+x=e^t$ when $x(0)=2, x'(0)=-1$. [NOV/DEC 2015, APRIL 2017] (8M).** BTL 3 **Answer: Refer Page No:5.103- Dr. G. Balaji.** Given:

	$x''(t)-2x'(t)+x(t)=e^t$ $x(0)=2; x'(0)=-1$ $[s^2L[x(t)]-sx(0)-x'(0)]-2[sL[x(t)]-x(0)]+L[x(t)]=L(e^t)$ $L[x(t)](s-1)^2=\frac{1}{s-1}+2s-2-3.$ $(3M)$ $L[x(t)]=\frac{1}{(s-1)^3}+\frac{2(s-1)}{(s-1)^2}-\frac{3}{(s-1)^2}$ $x(t)=L^{-1}\left[\frac{1}{(s-1)^3}\right]+2L^{-1}\left[\frac{1}{(s-1)}\right]-3L^{-1}\left[\frac{1}{(s-1)^2}\right]$ $=e^t\frac{t^2}{2}+2e^t-3e^tt$ $(5M)$
15	**Solve by using L.T$(D^2+9)y=\cos 2t$, given that if $y(0)=1, y\left(\frac{\pi}{2}\right)=-1$. [NOV/DEC 2004, MAY/JUNE 2009, APR/MAY 2015, DEC/JAN 2016] (8M)** BTL 3 **Answer: Refer Page No: 5.99- Dr. G. Balaji.** Given: $(D^2+9)y=\cos 2t.$ $y''(t)+9y(t)=\cos 2t.$ $L(y''(t))+9L(y(t))=L(\cos 2t).$ $[s^2L[y(t)]-sy(0)-y'(0)]+9L[y(t)]=\frac{s}{s^2+4}.$ $(2M)$ $(s^2+9)L[y(t)]=\frac{s}{s^2+4}+s+k.$ $L[y(t)]=\frac{s}{(s^2+4)((s^2+9)}+\frac{s+k}{(s^2+9)}.$ $L[y(t)]=\frac{1}{5}\frac{s}{s^2+4}+\frac{4}{5}\frac{s}{s^2+9}+\frac{k}{s^2+9}$ $(2M)$ $y(t)=\frac{1}{5}\cos 2t+\frac{4}{5}\cos 3t+\frac{k}{3}\sin 3t.$ $(2M)$ $\because y\left(\frac{\pi}{2}\right)=-1$ $\therefore y\left(\frac{\pi}{2}\right)=\frac{1}{5}\cos 2\left(\frac{\pi}{2}\right)+\frac{4}{5}\cos 3\left(\frac{\pi}{2}\right)+\frac{k}{3}\sin 3\left(\frac{\pi}{2}\right)=-1$ $k=\frac{12}{5}.$ $y(t)=\frac{1}{5}\cos 2t+\frac{4}{5}\cos 3t+\frac{4}{5}\sin 3t.$ $(2M)$